Optimal Dynamic Public Communication

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Abstract

This paper builds a dynamic model of the information flow between partially informed financial institutions and a public agency. The financial institutions decide how to allocate their portfolio between a riskless technology with known payoff and a risky technology whose payoff is unknown. The public agency learns about the value of the unknown payoff by observing with measurement error the actions of the financial institutions and decides on whether to communicate the information at the agency's disposal. The paper characterizes the optimal public communication plan and shows that full transparency (meant as revelation of information every period it is collected) is not always optimal. Instead, optimal plans involve delayed communication, the amount of delay depending in non trivial manners on the precision of private information and the size of the agency's measurement error. The reason for the result lies in the collection process of public information: while releasing information improves the welfare of the agents, it also decreases the informational content of their actions, hampering learning of the agency and reducing the benefits of future public communication.

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1 Introduction

Government authorities, like the Federal Reserve, the Bureau of Labor Statistics, often release documentation to the general public, might them be statistics or statements about current economic conditions. The aim of this public communication is, usually, to help decisions taking agents in situations when there is imperfect information about the state of the world.[1] As such, public communication is regarded as a welfare improving activity, and transparency, understood as the perfect communication of the knowledge government institutions have, is considered to be a guiding principle in the decision for releasing information for this type of institutions.

Recent body of research has been questioning the tenet of transparency.[2] The seminal paper of Morris and Shin (2002) identified a situation in which public communication was actually welfare detrimental. If individuals have complementarities in their payoff function, as in a beauty contest model in which each individuals would like to match what everybody else believes, then public communication can create over-reliance on the public information with respect to what is efficient. Amador and Weill (2010) present a model in which communication about monetary aggregates decreases the informational value of prices and might lead to welfare losses. Gala and Volpin (2010) show how public information, by inducing aggregate risk taking, may have negative welfare effects.

This paper explores a different analysis on the matter of public communication decisions with respect to previous work on two points. The first is that, while in previous work the source of public information is exogenous, this paper posits that governmental agencies learn about the state of the world by observing the actions of imperfectly informed agents, therefore it assumes endogenous public information.[3] This assumption is not far-fetched. Institutions like the Fed, the BLS, collect data, which are outcomes of the decisions of agents, analyze them and communicate their findings to the general public. More recently, in the wake of the financial crisis, the Obama administration instituted a Financial Stability Oversight Council, with the authority to *'collect information*

[1]There can be a different aim of public communication by governmental institutions, that is to help the public to correctly forecast the behavior of the institution. This is an important objective for Central Bank communication decisions, however it is not the object of analysis here. The two however can be connected, as further explored in the conclusions.

[2]Other than the cited work of Morris and Shin (2002), works questioning the positive welfare value of public communication are, among others, Morris and Shin (2005), Angeletos and Pavan (2007),Amador and Weill (2010), Gala and Volpin (2010), Lorenzoni (2010).

[3]If the object of information can be verified, then collection of information could be just based on direct reports. However, since verifiability does not often happen, there are large incentive for anyone to lie. Even if there are no incentives to lie, for instance when agents are infinitesimally small, then multiple equilibria could arise, in which agents are indifferent to lying or not.

from member agencies, other Federal and State financial regulatory agencies, the Federal Insurance Office and, if necessary to assess risks to the United States financial system, direct the Office of Financial Research to collect information from bank holding companies and nonbank financial companies'(Dodd 2010) and share this information with other authorities or the general public.

Collection of information and public communication become then intertwined processes. Present public communication can affect the choices of the agents in the economy through which the governmental agency will try to collect information in the future. If future agents' actions mostly reveal past public information, then learning of the agency will be minimal, influencing future public communication. The circularity of the process of collecting information and public communication can therefore have vicious effects in the learning process of agents about the true state of the world, undermining the objective of public communication as providing more information to the economy and so being welfare increasing.

The literature on informational cascades, for instance Banerjee (1992), already pointed out how public signals based on other individuals' choices can lead to inefficient decisions. Vives (1993) describes how public signals coming from prices slow the rate of learning of individuals. However, differently from market signals, which cannot be barred unless the market is shut down, public communication can be designed over time in such a way as to achieve maximum efficiency. The second novel approach of this paper with respect to the previous literature on information is to make public communication the explicit choice of an actor. If the ultimate objective of public communication is to be welfare improving for the agents, then we can ask the normative question of what is the optimal public communication plan. If a benevolent governmental agency has the power to decide whether they want to publicly release information or not, what is the welfare maximizing public communication strategy?

This paper provides an answer to this question in a specific setting, suggested by the Financial Stability Oversight Council introduced by the Financial Reform of the Obama Administration. The model describes the informational flow between privately informed financial institutions and a governmental agency that disseminates public information, and asks the normative questions: what is the optimal plan of public communication? Is it optimal to reveal information every time it is collected by the agency? In order to focus exclusively on the learning determined by public communication, there will be no markets in the model, and public communication will be the only source of learning for financial institutions in addition their private knowledge.

The main contribution of the paper is the characterization of optimal public communication plans. By first noting that it is always optimal to communicate at least in final periods, we are initially going to restrict the analysis to public communication plans that are monotonic, in the sense that might involve initial periods of silence, and after involve communication. We show that in this class of communication plans it is not always optimal to publicly communicate every time new information is collected by the governmental authority. Optimal communication plans can involve delay: governmental agencies can maximize welfare by deciding to not communicate until they have acquired enough knowledge, and then communicate.

The intuition for the result can be explained as follows. Financial institutions will base their choice on the private and public information available to them. When the agency observes the choices of financial institutions, it tries to extract their private knowledge, the fundamental source of learning for the agency. By communicating to the public, the agency induces actions that are based on public communication. Future observation of actions will reveal less private information to the agency and thus decrease the amount of learning by the agency. Therefore future public communication will be less informative. Having less information in any period is welfare decreasing for the financial institution since they cannot choose the correct investment. By not communicating in initial periods the agency assures that future public communication will be highly informative. Therefore delayed communication plans arise when the agency optimally trade-offs welfare benefits of communicating more information in the future versus the welfare costs of not communicating in initial periods. Delaying public communication can be welfare improving especially when financial institutions do not have precise knowledge of the state of the world. In determining the time of delay, two countervailing effects are important: the delaying effect determined by excessive reliance on public communication versus the anticipating effect determined by faster learning of the agency. The relative strength of the two pushes towards later or earlier revelation times.

We then enlarge the class of communication plans by allowing any arbitrary sequence of communication and silence periods in the plan. We show that enlarging the action space of the agency does not alter the optimal solution found before: optimal public communication plans always have a unique change of policy between not communicating and communicating. This results implies that once public communication starts, it is never optimal to stop communicating afterwards. For instance there is no advantage in providing the financial institutions with some preliminary information, temporarily increasing their welfare, then closing communication in order to let the agency efficiently learn, and then finally communicating. If conditions in the economy are such that it is optimal to have periods of silence in the communication plan, than it is most efficient to be silent at the beginning only.

The implication of the analysis is that transparency about fundamentals, understood as perfect communication of governmental agency knowledge, can lead to welfare losses. But while not communicating is never optimal, public communication can be timed in such a way as to provide information most efficiently to agents. In contrast to the literature on the social welfare value of information started by Morris and Shin (2002), the result is not based on socially inefficient complementarities in actions, but on the dynamic inefficiencies of public communication. Transparency is not welfare decreasing per se, but only because it prevents efficient use of the information in the economy. A governmental agency should therefore take into account the dynamic nature of information diffusion when thinking about release of information decisions, and realize that it might increase knowledge (and therefore welfare) by choosing an optimal timing for disclosure.

The structure of the paper is the following. Below we provide the closest related literature. Section 2 describes the model and explains the welfare trade-off generated by the public communication choice, while section 3 characterizes the optimal communication plan and comments the results. Section 4 concludes and explains further directions for research. The appendix provides all the proofs not given in the main text.

1.1 Related Literature

As discussed, this work is related to the literature on the social welfare value of information as started by Morris and Shin (2002). Their analysis showed that public communication can be welfare decreasing in the presence of complementarities of actions. Angeletos and Pavan (2007) extended the analysis to general quadratic utilities and showed that public information can have negative or positive effects depending on the actions being complementaries or substitutes. These models are all static, hence they abstract from the learning process. Also differently than these models, our setup is on the dynamic inefficiencies of public communication, not on payoff externalities.

Hellwig (2005) and Lorenzoni (2010) study implications of diffused imperfect knowledge and public signals for a monetary economy. Lorenzoni (2010) studies an economy in which individuals cannot distinguish between shocks to fundamentals and noise shock, and a monetary authority has to optimally set the policy rule. In this contest, increasing the precision of the public signal may lead to aggregate welfare losses if the central bank does not optimally change its policy in light of the new precision of the public signal. Differently than our work, knowledge is exogenous in these models, hence there is no feedback between public communication and learning.

In their long reflections on the value of central banks' transparency, Morris and Shin (2005) discuss a central feature of this work: a public communication regime may entail decreased precision of public information with respect to a not communicating regime when information is endogenous. Their analysis, even though based on a dynamic model, is on steady state properties of signals' precision, and misses the dynamic approach, the timing question and the welfare analysis present in this work.

Amador and Weill (2010) provides a different context in which public information can be welfare decreasing. When agents learn from the price system, the release of information about monetary shocks decreases the informational value of prices, leading to welfare losses. Their model is static and rooted in the substitutability of signals about different fundamentals of the economy, while mine analyzes communication only about one fundamental of the economy.

The social learning literature has analyzed the effects of the informational externality of public signals, starting with Banerjee (1992). This work shows that the presence of public communication may induce agents to disregard their private information and base their decisions only on public information. This creates informational cascades and herds which prevents agents to take the optimal action, decreasing welfare. However, the standard herding models are sequential move games, and the appearance of cascades and herds requires bounded beliefs and a discrete set of actions. My setup is characterized by unbounded beliefs and actions lie in a continuous space.

The works most related to this paper are Vives (1993), Vives (1997), and Amador and Weill (forthcoming). Vives analyzes a setup similar to mine, where agents learn from an aggregate public signal of each others' actions. Vives (1993) shows that the presence of the public signal slows down convergence of beliefs, while Vives (1997) determines welfare costs of the presence of the public signal. Amador and Weill (forthcoming) allows for the possibility of learning from public and private aggregates of each others' actions. They show that increasing the quality of initial public information may lead to negative welfare effects since it slows down the diffusion of the information through the agents. The main focus on these works is on channels of communication that operate through markets (prices) or through word-of-mouth, and not on governmental institutions' communication. My paper builds on these models but asks the different question of what is the optimal disclosure plan of public information.

Finally, this work is also related to the literature on information in financial markets, as in Grossman (1976), Hellwig (1980). Differently than this literature, the focus is not on information conveyed by prices but on information provided by public communication.

2 Model

The model represents the problem of a portfolio composition choice between a risky and a riskless technology, whose payoffs are R, known, and θ, unknown, respectively. These technologies are not publicly traded, hence there is no market mechanism determining prices. A setting without markets has the advantage to isolate the pure effects of public communication. In addition, this set up can also describe the situation when assets are traded in unregulated markets in which transaction prices are not public, and therefore do not constitute public signals about their returns.

The exposition of the model has five parts. The first part describes the problem of the financial institutions. The second part defines the private and public information available to the financial institutions and determines their expectations. The third part introduces the learning process for the agency. The fourth part defines the public message and the endogenous information structure of the economy. The fifth part states the problem of the agency of choosing the optimal public communication plan given the information structure.

2.1 The Investment Decision of Financial Institutions

In the economy there is a continuum $i \in [0,1]$ of financial institutions. Time is discrete and evolves from $t = 0, \ldots, T$. In each period, each financial institution faces the problem of choosing how to invest its wealth between two types of technologies, risky and riskless. The payoff of these technologies realize in period T only and are given by R for the riskless technology, and by θ for the risky technology.[4] The payoff of the riskless technology R is known, but the payoff of the risky technology θ is unknown by the financial institutions. At time $t = 0$ all financial institutions have a common prior over the payoff θ given by $N(\bar{\theta}, 1/P_\theta)$.

The value of θ characterizes the state of the world in period T, as such, the most gen-

[4]Investment options of financial institutions are usually more varied than just two types of technologies (assets). The model can be easily extended to incorporate multiples technologies, for instance, see Admati (1985)

eral real world counterpart of θ is productivity over the business cycle,[5] assumed to affect the returns of the technology. This is also the most direct interpretation in terms of communication of public agencies. But θ can also represent a shock to returns idiosyncratic to the particular technology used. Idiosyncratic elements can be specific technical characteristics of the technology (asset), like the exact definition of contract clauses for complex securities. Public agencies that have the role to regulate markets are often involved in communication over such matters.

Each financial institution is endowed at the beginning of time with some wealth already invested in the two technologies, $B_{i,-1}$ for the riskless technology and $D_{i,-1}$ for the risky technology. In any period $t = 0, 1, \ldots, T$, the financial institution can choose whether to change its position or not. If it does, it has to pay an adjustment cost. At any time t the budget constraint is given by:

$$B_{i,t-1} + D_{i,t-1} = B_{i,t} + D_{i,t} + \frac{\lambda}{2}(D_{i,t} - D_{i,t-1})^2 \tag{1}$$

where prices have been normalized to one and $\lambda > 0$ parametrizes the adjustment costs. The adjustment costs are directly related to the change in quantity of the risky technology only. This assumption implies that costs determined by changing the quantity of risky technology in a portfolio are first order with respect to costs determined by the change in the riskless technology. For instance, if we consider $D_{i,t}$ as loans originating in the financial institution, than there are front-office and back-office real costs that the financial institution has to bear, such as interacting with the customers, determining the quality of the loan, and so on. These costs are relatively greater than those a financial institution would face when changing the quantity of wealth invested in riskless assets, where the process is more streamlined (attending government bond auctions) and smaller real costs have to be borne.

The financial institutions are risk neutral and want to maximize wealth at the end of period T, when payoffs of the technologies are realized. Given its position in the two technologies $B_{i,T}$ and $D_{i,T}$ in final period T, wealth will be given by:

$$W_{i,T} = B_{i,T}R + D_{i,T}\theta \tag{2}$$

In any period t the financial institution i chooses $B_{i,t}$ and $D_{i,t}$ with the objective of maximizing its expected wealth $W_{i,T}$ subject to the budget constraint (1). By recursively sub-

[5] Miccoli (2010) studies in a similar setup the case when θ is evolving over time as a random walk.

stituting $B_{i,t}$ from the budget constraint, its final wealth is also equal to:

$$W_{i,T} = B_{i,-1}R + D_{i,-1}\theta + (\theta - R)\sum_{t=0}^{T} x_{i,t} - \frac{\lambda R}{2}\sum_{t=0}^{T} x_{i,t}^2 \tag{3}$$

where $x_{i,t} \equiv D_{i,t} - D_{i,t-1}$. Since in period t $D_{i,t-1}$ is predetermined, we can represent the choice of $D_{i,t}$ by $x_{i,t}$, that is the change over last period technology holdings. Hence we can rewrite the financial institution's problem as:

$$\max_{\{x_{i,t}\}_{t=0}^{T}} \mathbb{E}_{i,0}\left[B_{i,-1}R + D_{i,-1}\theta + (\theta - R)\sum_{t=0}^{T} x_{i,t} - \frac{\lambda R}{2}\sum_{t=0}^{T} x_{i,t}^2\right]. \tag{4}$$

The first order condition determining the change of position in the risky technology is given by:

$$x_{i,t} = \frac{\mathbb{E}_{i,t}[\theta] - R}{\lambda R}. \tag{5}$$

Two comments are necessary in order to explain how the models works. First, this model allows for short-sale. This is a common feature of finance models with information[6]. Second, the financial institution wants to change its composition of investment depending on whether it thinks that $\mathbb{E}[\theta]$ is greater or smaller than R. For instance, if it believes $\mathbb{E}[\theta] > R$, it would like to invest everything in the risky technology. However it cannot fully adjust in one period given the presence of the adjustment costs, hence it will keep on changing every period their portfolio allocations until it reaches their optimal composition. Without loss of generality, from now on, λ and R will be both normalized to 1.

2.2 Information: Financial Institutions

Apart from the initial wealth endowment, in the previous section financial institutions are all symmetrical. Information private to the financial institutions about the final payoff, θ, is what introduces asymmetries in the model.

In every period t, financial institutions have two sources of information about the payoff θ: private and public. The private source of information is a private signal about θ:

$$\theta_i = \theta + \eta_i \tag{6}$$

[6] As for instance in the basic CARA-Normal setup introduced by Grossman (1976)

where $\eta_i \sim N(0, 1/p)$, independent and identically distributed (i.i.d) in the cross-section of the economy. p, the inverse of variance of the η_i, defines the precision of the signal conditional on θ, and, as is it customary in the literature, we will measure information in units of precision.

The second source of information comes from public signals, observed by all financial institutions. In particular, in every period t the financial institutions receive, sent by an agency introduced below, a public signal about θ defined by $\Theta_t(\theta)$. Financial institutions keep this information for the future and therefore, at any point in time t, financial institutions have access to an history of public signals about θ indicated by $\Theta^t = \{\Theta_0(\theta), \Theta_1(\theta), \ldots, \Theta_t(\theta)\}$. The origin and structure of this public signal are endogenous in the model, and will be fully characterized below. The only conjecture needed now is that the public signals are, conditional on θ, normally distributed, independent among them and of θ_i. This conjecture will be proved true below.

It is convenient to study separately the information coming from public sources and the information coming from private ones. Given a history of public signals about θ, the financial institutions in any period t observe a new public signal, and update their beliefs determined by public signals only by using Bayes' Rule in order to form a posterior over the distribution of θ. Since signals are normally distributed and independent, the conjugate, the distribution describing the posterior, will also be normal, and it is characterized by:

$$\mu_t^P = \mathbb{E}[\theta|\Theta^t], \tag{7}$$

$$P_t = \mathbb{V}ar_t^{-1}[\theta|\Theta^t] \tag{8}$$

where μ_t^P is the mean and P_t is the precision of information determined by public signals. The normality assumption implies that the mean of the posterior is also normally distributed and it is a sufficient statistics for the public information.

Financial institutions use private and public information to determine their posterior beliefs on θ. In any period t, using standard formulas for Bayesian updating with, conditional on θ, normal and independent signals, the mean posterior beliefs of financial institutions about θ will be a convex combination of private and public signals, with weights determined by their relative precisions, and the total precision will be given by the sum of the precisions of public and private information, that is:

$$\mathbb{E}_{i,t}[\theta] = \frac{p}{p + P_t}\theta_i + \frac{P_t}{p + P_t}\mu_t^P, \tag{9}$$

$$\mathbb{V}ar_t^{-1}[\theta] = p + P_t. \tag{10}$$

Note that the precision of beliefs does not depend on financial institutions' specific information, since the precision of private knowledge is common across financial institutions. What is different among them is the expectation, as given by their private information θ_i. The term $\frac{p}{p+P_t}$ determines the weight financial institutions place over their private information. This is decreasing in P_t: the higher is P_t, the more precise is public information, the less financial institutions are going to use their private knowledge when forming their beliefs. Conversely, it is increasing in p: higher precision of private information implies a higher reliance on the private signal.

2.3 Information: Agency

In the economy there is a public authority, generically called agency, that has the power to observe in each period the investment decisions of the financial institutions, and to send, in each period, a public signal to them, determining the public information available to the financial institutions. The agency's knowledge derives from a learning process. In the initial period, the agency does not have superior information about θ then the financial institutions, in particular, it shares the same common prior $\theta \sim N(\bar{\theta}, 1/P_\theta)$ with all the financial institutions.

The agency's learning process is through the observation of the actions of financial institutions. In particular, the agency has the power to collect information about θ by observing, with some noise, the change in the financial institutions' portfolio composition. Formally, in any period t the agency observes a signal:

$$S_{i,t} = x_{i,t} + \varepsilon_t \tag{11}$$

where $\varepsilon_t \sim N(0, 1/\tilde{p}_\varepsilon)$, i.i.d. over time and independent of η_i, represents the measurement error, aggregate and not financial institution dependent, of the collection process of the agency. Given the optimal choice $x_{i,t}$ of the financial institutions, the signal $S_{i,t}$ reveals the beliefs of the financial institution i:

$$S_{i,t} = \frac{p}{p+P_t}\theta_i + \frac{P_t}{p+P_t}\mu_t^P - 1 + \varepsilon_t. \tag{12}$$

Since the agency is the source of public information, all the information contained in μ_t^P is already part of its information set at time t, and the only relevant information it can extract from observing the change in financial institutions portfolios is the one given by private information. Therefore by aggregating the signals coming from all financial institutions and by eliminating the already known μ_t^P, the agency observes every period a signal:

$$S_t = \theta + \frac{p+P_t}{p}\varepsilon_t, \tag{13}$$

where we have used the convention that the laws of large number holds, so that $\int \eta_i di = 0$ almost surely. Note that the precision of S_t conditional on θ is given by $\left(\frac{p}{p+P_t}\right)^2 \tilde{p}_\varepsilon$. There are therefore two factors that affect the quantity of information the agency can extract from its signal. The first one is the precision of the measurement error, $\tilde{p}_\varepsilon$. If in the limit the agency were to be able to get rid of the noise in its signal, then it would observe perfectly θ by aggregating the information of the financial institutions. But if the precision $\tilde{p}_\varepsilon$ is bounded away from infinity, then perfect observation of θ is never achieved.

The second factor that determines the precision of the information the agency receives is the weight financial institutions place on their own private information when forming their beliefs, $\frac{p}{p+P_t}$. The higher the relative precision of private information with respect to public information, the more useful is the information the agency can extract from the financial institutions. This factor will be key in determining the welfare values of public communication.

Note that S_t changes over time by the realization of the measurement error ε_t, but more importantly by the contemporaneous level of public knowledge P_t. Therefore the evolution of public knowledge over time determines the quality of the signal observed by the agency. We are going to define $S^t(P^t) = \{S_0(P_0), \ldots, S_t(P_t)\}$ to be the history of signals the agency has received at end of any period t.

2.4 Public Communication and Endogeneity of Public Beliefs

We are going to assume that, at the beginning of every period t, the agency can choose either to not communicate anything, or to communicate its mean beliefs using the information at its disposal. Since this is done before financial institutions make their choice, the mean beliefs is conditional on the history of observed signals by the agency $S^{t-1}(P^{t-1})$.[7]. Formally:

$$\Theta_t = \begin{cases} \emptyset \\ \mathbb{E}[\theta | S^{t-1}(P^{t-1})] \end{cases} \qquad \forall t, \tag{14}$$

where $\emptyset$ defines the no message choice, and $\mathbb{E}[\theta|S^{t-1}(P^{t-1})]$ defines the mean posterior belief of the agency conditional on the history of signal observed. Note that by explicitly defining the public message we have also implicitly proven the conjecture made above about its distribution and its independence from the private information. Given that the public signal is the mean of the posterior beliefs of the agency, this object is normally distributed, and its noise terms ε_s, which are the aggregate measurement error of the signals received by the agency, are independent of η_i, the noise realization in private information. Therefore public and private information are, conditionally on θ, independent.

Public communication decisions affect in two fundamental ways the beliefs of the financial institutions. Firstly, any decision to communicate or not in period t directly influences the beliefs of the financial institutions for period t, as equations (7), (8) show. Secondly, the communication decision, through the precision of beliefs of the financial institutions for period t, also affects the information the agency receives in the future (since $S^t(P^t)$), and hence by how much the agency can influence through public communication the beliefs of the financial institutions in the future.

The key element that public communication influences is the precision of beliefs P_t. As we have seen in equation (8), $P_t(\Theta^t)$. On the other hand the amount of information available to the agency, and so its public message if sent, is dependent on history $S^{t-1}(P^{t-1})$, that is $\Theta_t(P^{t-1})$. A public communication *plan*[8] $\{\Theta_t\}_{t=1}^T$ determines a relation between past and present precision of public information P_t. The exact relation will be defined below.

The evolution of public information beliefs is therefore endogenous in the economy. This is true not only because public knowledge is determined by the decision to communicate or not by the agency. But also because the same information that can be released by the agency depends on its past communication decisions. Or, equivalently, communi-

[7]This assumption is made without loss of generality. The same results would be obtained if the agency would send the public signal in any period t after the financial institutions had made their choice. Given that financial institutions have memory, they would use this signal next period when making their choice. This would amount only to a change of time indices.

[8]I will use plan when talking about the problem of the agency, and history when talking about the realization of the plan.

cation decisions today affect what the agency will observe and know in the future.

2.5 The Agency's problem

The objective of the agency is to choose, at time 0, a public communication plan $\{\Theta_t\}_{t=1}^{T}$[9] that maximizes ex-ante aggregate welfare.[10] The particular plan chosen will determine the endogenous precision of the public knowledge in any period t and the way financial institutions are going to use the realization of public and private information for determining their expectation and choices. Since the agency chooses at time 0 the plan of public communication we are assuming that the agency can credibly commit to its plan.

The ex-ante aggregate welfare function is given by the following proposition:

Proposition 1 (Aggregate ex-ante Welfare)**.** *Aggregate ex-ante welfare is given by:*

$$W(T) = \kappa - \frac{1}{2}\sum_{t=0}^{T}\frac{1}{p+P_t} \tag{15}$$

where $\kappa \equiv D_{-1}\bar{\theta} + B_{-1} + \frac{T+1}{2}\left[(\bar{\theta}-1)^2 + \frac{1}{P_\theta}\right]$ *and* B_{-1} *and* D_{-1} *represents, respectively, the initial aggregate endowment of risky and riskless technology.*

Proof. In the appendix. □

The linear-quadratic setting with the normality assumption allows us to derive an ex-ante aggregate welfare function which depends only on the precision of the information and not on the realizations of the signals. Aggregate ex-ante welfare is increasing in total precision of beliefs $p + P_t$ in period t, that is, having more precise information is beneficial to the financial institutions in any period. This is because not knowing θ is costly for the financial institutions, given the adjustment costs of changing their position.

The object of this work is to analyze the problem of the agency that has to decide its optimal public communication plan with the object of maximizing aggregate ex-ante welfare subject to the endogenous evolution of precision of public information in the

[9] Note that public communication in $t = 0$ is not meaningful since the only information the agency can communicate is the initial prior, common however to all financial institutions.

[10] There are no consumers in this model, hence the use of aggregate welfare of the financial institutions as the objective of the agency might seem arbitrary. However we can justify it if we think of a general equilibrium setting in which consumers own shares of the financial institutions.

economy, that is:

$$\max_{\{\Theta_t\}_{t=1}^{T}} \left[\kappa - \frac{1}{2}\sum_{t=0}^{T} \frac{1}{p+P_t}\right]$$
$$\text{s.t.} \quad (8), (13), (14).$$

2.6 The trade-off between present and future public communication

The potential source of problem with public communication lies in the dynamics of the information flow between the financial institutions and the agency. By communicating in period t the information at its disposal the agency provides more knowledge about θ to the financial institutions (P_t increases), increasing period t contribution to welfare, and since financial institutions have memory, also increases contributions to welfare in future periods. However remember that the precision of the information the agency extracts every period from the financial institutions is given by

$$\mathbb{V}ar^{-1}[S_t|\theta] = \left(\frac{p}{p+P_t}\right)^2 \tilde{p}_\varepsilon \tag{16}$$

and this is decreasing in P_t. Public communication in any period t, by increasing P_t, will imply a less precise signal to the agency and therefore lower informational content of public communication in the following periods. The dynamic creates a negative feedback between precision of public communication today and tomorrow, present communication dampening the precision of future communications. This implies that welfare in future periods can increase if no public signal is sent in period t conditional on communicating some time in the future: the agency can acquire more precise knowledge and communicate more information in the future which will determine higher welfare benefits than just communicating every period. However the future welfare benefits have to be weighed against the welfare losses of not communicating in period t. Therefore even if present public communication always dampens the effect of future public communication on welfare, not always by withdrawing present public communication higher efficiency can be achieved.

The reason for the potential efficiency loss of public communication is the fact that financial institutions do not internalize future costs determined by underproduction of public information. When a single financial institution receives the public signal, by relying onto it in a way proportional to its precision, decreases the informational content of future public communication. However this behavior is optimal for the financial institutions: given their infinitesimal size, each financial institution cannot on its own influence the precision of the information collected by the agency. As in Vives (1997), public signals can therefore create an *informational externality*.

The potential trade-off between present and future public communication underscores the fundamental question of this paper: how can public communication be designed in such a way as to achieve maximum efficiency? Given the potential negative effect in the future of public communication, maximum efficiency can be achieved by appropriately choosing the timing of the public information release: when is public information most advantageous to the financial institutions?

3 Optimal Public Communication

Given the nature of the trade-off, it is clear that the agency would want to communicate for sure at least in the last period. Sending a public signal in the last period improves the welfare of the agency and does not create any negative feedback on learning, since at the end of the period the payoff θ of the risky asset is revealed. This is the initial characterization of optimal public communication plan, stated in the following proposition.

Proposition 2. *Optimal public communication plans always involve communication at least in period T.*

Proof. In the text. □

3.1 Delayed Public Communication Plans

Given proposition 2, we are first restricting the analysis to communication plans that involve silence for some period and then communication, therefore the case of *delayed* public communication. This is a strong limitation on the action space of the agency. However, later on we will relax this restriction and we will show that the optimal communication plan chosen in this smaller action space remains the same even when we allow for a more general choice set. In this section the agency has to choose $\tau \in [0, \ldots, T]$ such that:

$$\Theta_t = \begin{cases} \emptyset & t < \tau \\ \mathbb{E}_{A,t}[\theta | S^{t-1}(P^{t-1})] & t \geq \tau \end{cases} \tag{17}$$

where $\emptyset$ defines no public signal and $\mathbb{E}_{A,t}[\theta|S^{t-1}(P^{t-1})]$ are the mean beliefs of the agency conditional on the information at its disposal.[11] Up to period τ the agency is being silent, then it starts revealing the mean of its beliefs. When communicating the agency is fully transparent: every new information received will be communicated to the public next period together with also all information received in the past.

The definition of the communication structure allows us to explicit the endogenous precision of public information as described by equations (8). This is done in the following proposition.

Proposition 3. *The precision of public knowledge P_t, given the delayed communication plan of the agency (17), is:*

$$P_t = \begin{cases} P_\theta & t < \tau \\ P_\theta + \tau \left(\frac{p}{p+P_\theta}\right)^2 \tilde{p}_\varepsilon & t = \tau \\ P_{t-1} + \left(\frac{p}{p+P_{t-1}}\right)^2 \tilde{p}_\varepsilon & t > \tau \end{cases} \tag{18}$$

Proof. In the text. □

When $t < \tau$ there is no public signal, public precision is only the precision of the initial prior the financial institutions have, P_θ.[12] When $t = \tau$ the Agency sends the first public signal, which is the mean of its belief. Given that the agency has observed τ signals, each conditional on θ normally distributed with precision $\left(\frac{p}{p+P_\theta}\right)^2 \tilde{p}_\varepsilon$, its mean beliefs are an equally weighted convex combination of signals S_t. The public signal in τ is hence given by:

$$\Theta_\tau = \mathbb{E}_{A,\tau-1}[\theta] = \frac{\sum_{s=0}^{\tau-1} \left(\frac{p}{p+P_\theta}\right)^2 \tilde{p}_\varepsilon S_s}{\sum_{s=0}^{\tau-1} \left(\frac{p}{p+P_\theta}\right)^2 \tilde{p}_\varepsilon} = \frac{\sum_{s=0}^{\tau-1} S_s}{\tau} = \theta + \frac{1}{\tau}\frac{p+P_\theta}{p}\sum_{s=0}^{\tau-1} \varepsilon_s.$$

The measurement error realizations ε_t are i.i.d., therefore we can just sum the precision of the signals received by the agency to determine the precision of the public signal: this given by $\mathbb{V}ar_\tau^{-1}[\Theta_\tau|\theta] = \tau \left(\frac{p}{p+P_\theta}\right)^2 \tilde{p}_\varepsilon$. Since both the initial prior and the signal are normally distributed then precision of public knowledge in τ is given by the sum of their precisions.

For $t > \tau$, the Agency communicates its mean beliefs every period, however the only new information in the public signal is given by the new signal the Agency has received

[11] Note that the normality of signals implies that the mean is a sufficient statistics for the history of signals observed by the agency

[12] Since the initial prior is common to all financial institutions, we consider it part of the public information.

in the last period. This is a normally distributed signal, and the precision of this new information is given by: $\left(\frac{p}{p+P_{t-1}}\right)^2 \tilde{p}_\varepsilon$ (this is the precision of the signal S_{t-1} the agency observes in period $t-1$, which constitutes the new information the agency is communicating in period t). The financial institutions use both previous public information and the new information coming from the public signal to determine the precision of the public information beliefs at time t. Since both are normally distributed, the posterior precision will be given by the sum of the two, which determines the recursion of public information precision when the agency is communicating.

3.2 Optimal Delayed Public Communication

Even though the model has been described in discrete time, in order to prove the results it is more convenient to switch to a continuous time approximation. This approximation allows us to obtain a closed form equation for the evolution of precision and to use standard maximization techniques to solve our problem. The approximation is obtained by letting the observational noise of the signal to agency grow very large while at the same time letting the agency observe the signal more and more often.[13]

Proposition 4. *Let Δ be the time interval, and assume that $\tilde{p}_\varepsilon(\Delta)/\Delta \to p_\varepsilon$ as $\Delta \to 0$. Then the continuous time version of evolution of public knowledge is given by:*

$$\dot{P}(t) = \left(\frac{p}{p+P(t)}\right)^2 p_\varepsilon \tag{19}$$

Proof. Rearrange the evolution of public precision as:

$$\frac{P_{t+\Delta} - P_t}{\Delta} = \left(\frac{p}{p+P_t}\right)^2 \frac{\tilde{p}_\varepsilon(\Delta)}{\Delta} \tag{20}$$

By taking the limit as $\Delta \to 0$ the result obtains. □

The equation in (19) is an ordinary differential equation that, together with the boundary condition $P(\tau) = P_\theta + \tau \left(\frac{p}{p+P_\theta}\right)^2 p_\varepsilon$, has a unique solution given by:

$$P(t) = -p + (p + P_\theta)\left[(1+\alpha\tau)^3 + 3\alpha(t-\tau)\right]^{1/3} \tag{21}$$

[13]The continuous time approximation determines that the signal the agency observes is described by a stochastic differential equation $dS_t = x_t dt + \frac{dW}{\sqrt{p_\varepsilon}}$, where W is a Weiner process, and $x_t = \int x_{i,t} di$.

where $\alpha \equiv \frac{p^2}{(p+P_\theta)^3} p_\varepsilon$. Equation (21) describes the evolution of precision of public knowledge at any point in time after the communication decision. Note that, as in Vives (1993) (with discrete time) and Amador and Weill (forthcoming) (with continuous time) convergence of beliefs obtains at rate $t^{1/3}$.[14] This is lower than the linear rate it would obtain if financial institutions were observing every period exogenous i.i.d. signals. The lower convergence rate is the result of financial institutions observing aggregate information of their decisions, as Vives (1993) showed. The continuous time version of the problem of the agency is:

$$\begin{aligned}
&\max_{\tau \in [0,T]} \left[\kappa - \int_{t=0}^{T} \frac{1}{p + P(t)} dt \right] \\
&s.t. \quad P(t) = P_\theta \quad t < \tau \\
&\qquad P(t) = -p + (p + P_\theta) \left[(1 + \alpha\tau)^3 + 3\alpha(t - \tau) \right]^{1/3} \quad t \geq \tau
\end{aligned} \tag{22}$$

Note that there is no assurance of concavity of the problem in τ for all parameter values, however maxima (the solution need not be unique) will always exist by the extreme value theorem, since one can show by substituting the constraint and solving the integral that the objective function is continuous over the closed and bounded interval of τ.

Before turning to the main results, the following lemma illustrates the effect of delaying public communication onto the evolution of precision of knowledge.

Lemma 1 (Evolution of Precision with Delayed Communication)**.** *Let $P(t) + p = (p + P_\theta) \left[(1 + \alpha\tau)^3 + 3\alpha(t - \tau) \right]^{1/3}$ denote the precision of beliefs under a delayed communication plan for $t \geq \tau$ (as in equation (21)), and $P'(t) + p = (p + P_\theta) [1 + 3\alpha t]^{1/3}$ denote the precision of beliefs when agency always communicates ($\tau = 0$). Then, for all $\tau > 0$, $P(t) + p > P'(t) + p$ for $t \geq \tau$.*

Proof. In the appendix. □

Figure 1 graphically shows the result of lemma 1 for a given τ. The bold black line shows the evolution of the precision with delayed public communication, while the thin gray line shows the evolution when the agency starts communicating at the beginning. Clearly before τ the delayed public communication plan does not improve knowledge of the financial institutions. At τ, when the agency starts communicating, precision under

[14] Note that, when the agency is communicating, it is as if the financial institutions were observing a signal about aggregate choices. Thus the similarity with Vives (1993) and Amador and Weill (forthcoming) setup and the same rate of convergence of public beliefs.

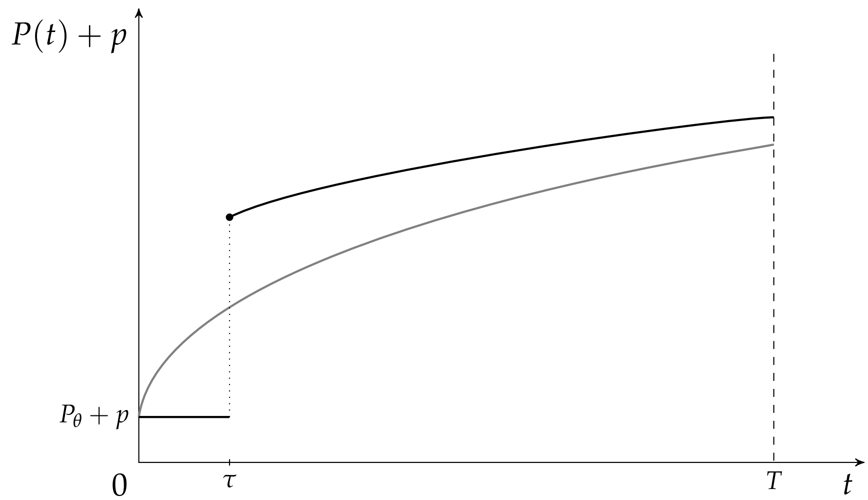

Figure 1: Evolution of precision of knowledge

the delayed communication plan jumps higher than precision under the always communicating plan. The intuition is clear: by not communicating in the interval $[0, \tau)$ the agency has been able to observe more precise information about the state of the world with respect to the always communicating plan. Remember from equation (16) that the precision of the signal to the agency is higher when precision of public knowledge is lower. Since the precision of public knowledge when there is no public communication is lower, when the agency starts communicating in period τ it has observed more precise signals with respect to the always communicating plan, therefore precision of public knowledge in τ is higher. After τ precision of knowledge under both plans increases at rate $1/3$, hence precision under the always communicating plan will never catch up in finite time: precision under the delayed communication plan will always be constantly higher than under the always communicating plan.

The evolution of precision of public information directly translates into welfare, since higher precision implies higher welfare. The choice of a delayed communication plan is welfare improving with respect to an always communicating plan if the welfare benefits given by having more knowledge after τ more than offset the welfare costs of not communicating before τ. If so, by choosing the appropriate τ the agency can effectively design public communication in order to achieve maximum efficiency.

The following theorem is one of the main contributions of the paper and characterizes the optimal public communication plans given the welfare trade-offs between having more information now or in the future.

Theorem 1 (Delayed Optimal Communication)**.** *Consider the welfare maximization problem* *under the delayed maximization plan defined in (22). Then:*

i. The problem admits unique solution τ^*;

ii. Let $\alpha \equiv \frac{p^2}{(p+P_\theta)^3} p_\varepsilon$. For any $T > 0$, $\tau^* > 0$ if and only if $\alpha > \frac{c}{T}$, $c \in \mathbb{R}^{++}$. For $\alpha \leq \frac{c}{T}$, $\tau^* = 0$.

Proof. In the appendix. □

The theorem states that optimal public communication plans can involve delay, a period during which the agency does not communicate to the public but only learns. The intuition for why delaying public communication can be optimal has already been given: not providing a public signal induces more efficient learning of the agency and hence a more precise future public communication. By delaying communication the agency is effectively making the financial institutions internalize the costs of under-production of information, in this way it solves the informational externality. Once the agency has acquired enough knowledge then communication can start.

The intuition for when delaying public communication is optimal can better be expressed by switching back to the discrete time case. Consider the evolution of public knowledge precision when the agency is communicating given in proposition (3), then:

$$\frac{\partial P_{t+1}}{\partial P_t} = 1 - 2\frac{p^2}{(p+P_t)^3}\tilde{p}_\varepsilon. \tag{23}$$

When increasing public precision in any period t by communicating there are two effects happening. One is a positive *memory effect* (the first term in (23)): more public information today implies more public information tomorrow, since financial institutions carry information from the past. The other is a negative *jamming effect* (the second term in (23)): more public information today decreases the informational value of future public communications, in a way public information *jams* the informativeness value of future communications. The jamming effect is the result of the financial institutions relying on the public communication channel. If the jamming effect is sufficiently large, then communicating has negative effect on tomorrow's welfare. In such a situation public communication implies that present and future precision of knowledge move in opposite directions: by withholding public communication, the agency is able to increase tomorrow's precision, it is unjamming the public communication channel. Hence the optimality of the policy. Since in our problem the question is not whether to increase the precision of public communication but whether to communicate or not, $\alpha = \frac{p^2}{(p+P_\theta)^3} p_\varepsilon$ represents the jamming effect when public communication has not yet happened, and therefore public information in the economy is nothing else then the precision of the initial prior P_θ.

However, the jamming effect on its own is necessary but not sufficient to imply optimal delayed communication plans. What is also needed is that welfare losses of not communicating for some periods are compensated. This is the role of the threshold $\frac{c}{T}$. A longer time horizon implies that the informational benefits of delaying public communication last for longer periods, therefore the threshold decreases, i.e. delayed public communication is optimal for a larger area in the parameters' space.

Note that α is decreasing in the precision of knowledge of the financial institutions $(p + P_\theta)$. Hence when the financial institutions have very little knowledge about θ, the jamming effect determined by public communication will be stronger. In this instances delaying communication, even if hurting in the short terms the financial institutions, will be able to let the agency efficiently accumulate information and achieve higher overall welfare.

3.3 Optimal Generic Public Communication Plans

So far the action space of the agency has been limited to deciding when to start communicating only once. We are now going to relax this assumption, allowing for an arbitrary finite number of periods of communication or silence by the agency, and we will let the agency choose the length of each.

Given proposition 2, any communication plan will involve communication at least in the last instant. Also, if the agency ever wants to be silent, it will be in the beginning, since it is when the marginal benefits of not communicating will be highest, given that the time horizon is the longest. Therefore an optimal communication plan will surely imply communication in a neighborhood of $t = T$ and might entail silence in a neighborhood of $t = 0$.

Let $C_{2n+1} = \{\tau_1, \tau_2, \dots, \tau_{2n+1}\}$, $n \in \mathbb{N}$, such that $0 \leq \tau_1 \leq \tau_2 \leq \cdots \leq \tau_{2n} \leq \tau_{2n+1} < T$, indicate the set of cut-points of the communication plan of the agency, i.e. the times when the agency switches from communicating to silence or viceversa. Since optimal communication plans might entail silence but will surely end with communication, we consider plans that start with the possibility of silence and end with communication, hence the even number of intervals (or the odd number of elements in the set). The agency is silent in $[0, \tau_1)$, communicates in $[\tau_1, \tau_2)$, is silent again in $[\tau_2, \tau_3)$, and so on.[15] The intervals in

[15]The assumption that the communication plan starts with a silence period is without loss of generality,

which the agency is communicating have as lower bound an odd index number, i.e. are of the type $[\tau_{2k+1}, \tau_{2(k+1)})$. Conversely the agency is not communicating in the intervals $[\tau_{2k}, \tau_{2k+1})$. The set C_{2n+1}, together with assumption that $\Theta_t = \emptyset$ for $t \in [\tau_{2k}, \tau_{2k+1})$ and $\Theta_t = \mathbb{E}_{A,t}[\theta]$ for $t \in [\tau_{2k+1}, \tau_{2(k+1)})$, for all $k \leq n$, where $\tau_0 \equiv 0$ and $\tau_{2(n+1)} \equiv T$, fully define a generic communication plan of the agency parametrized by n. With an abuse of notation we will also indicate as $W : \mathbb{R}^{2n+1} \to \mathbb{R}$ to be the ex-ante aggregate welfare as function of C_{2n+1}. The problem of the agency is therefore to $\max_{\{\tau_j \in C_{2n+1}\}_{j=1}^{2n+1}} W(C_{2n+1})$ subject to the endogenous evolution of precision of information. The following proposition describes the optimal public communication plan in a generic action space.

Theorem 2 (Optimal communication in general action space)**.** *Let $W(\tau^*)$ indicate the maximum ex-ante aggregate welfare achievable under the delayed communication plan, where $\tau^* = \arg\max_{\tau \in [0,T]} W(\tau)$. Then, for all $n \geq 1$, for all $\tau_i \in C_{2n+1}$, $i = 1, \ldots, 2n+1$, $W(\tau^*) \geq W(C_{2n+1})$, with equality if and only if, for any $0 \leq k \leq n$, $\tau_{2k+1} = \tau^*$, $\tau_{2(j+1)} - \tau_{2j+1} = 0$ for all $j < k$ and $\tau_{2j+1} - \tau_{2j} = 0$ for all $j > k$.*

Proof. In the appendix. □

The theorem states that it is not possible to achieve strictly higher welfare by using a communication plan that is different from the delayed communication plan. Therefore optimal public communication plans always involve a unique bang-bang behavior: communication goes from nothing to everything, but it never reverts back.

This result is somewhat striking, since one could imagine that releasing an initial amount of information, in order to improve at the beginning the welfare of the financial institutions when they need it most, then shutting down communication in order to efficiently collect information and finally communicating again, might lead to a higher overall welfare. This is indeed not the case, and any communication plan with an arbitrary number of communication and silence intervals, each of any length, will not give higher welfare then the simple one cut-point plan.

The intuition for the proof is based on two observations. The first one is that the problem in the general action space can actually be decomposed into smaller problems by recursively choosing the length of the last communication period given all other intervals. The problem becomes quite tractable because the precision of public information at any point in time is the only information needed for choosing deciding over the next communication spell. For instance, if the agency wants to choose $\tau_{2n+1} \in [\tau_{2n}, T)$ the only relevant information is the precision of public information $P(\tau_{2n})$ and the length of

since the agency has always the option to set $\tau_1 = 0$ and therefore to start with communication.

the interval over which we are maximizing, $T - \tau_{2n}$. This problem is isomorphic to the problem presented in proposition (1), but a new threshold rule will apply here, one based on the amount of public information available in τ_{2n}. If $\frac{p^2}{(p+P(\tau_{2n}))^3} p_\varepsilon > \frac{c}{T-\tau_{2n}}$, then it will be optimal to set $\tau_{2n+1} > \tau_{2n}$, otherwise the agency is always communicating in the interval $(\tau_{2n}, T]$. In this way the problem can be solved recursively from the choice of the last cutpoint, similar to a backward induction solving method.

The second, and most important observation, is that there is monotonicity in the behavior of the constraint. Consider the constraint at some point in time $\tau_j \in [0, T]$. The jamming effect is given by $\frac{p^2}{(p+P(\tau_j))^3} p_\varepsilon$. By going backwards then, for any communication strategy, $P(\tau_j)$ can only weakly decrease, making the jamming effect weakly increase. On the other hand the threshold $\frac{c}{T-\tau_j}$ decreases as τ_j decreases. Therefore if the constraint binds at some point in time, for sure it binds also in previous points. Said differently, the constraint can only be binding for some compact interval $[0, \bar{\tau}] \subseteq [0, T]$

The two observations imply that when the agency is choosing its communication plan, if $\bar{\tau} = 0$, the constraint never binds, the jamming effect is never strong enough then the optimal plan involves always communicating. If $\bar{\tau} > 0$, then the agency will choose to always communicate for all choices in $[\bar{\tau}, T]$, and in $[0, \bar{\tau}]$ by recursively solving backwards, it will iteratively eliminate communication/not communication periods until the optimal communication plan involves the unique cut-point τ^*.

3.4 Comparative Statics

The choice of when to start communicating, τ^*, will be determined by equating the marginal costs of being silent for one more unit of time versus its marginal benefits. The marginal costs are clear: an additional period of silence implies an additional period in which the financial institutions have only their private information and prior to determine the value of θ, therefore an additional period in which utility is given by $\frac{1}{p+P_\theta}$. Marginal benefits have a less straightforward formulation, however they characterize the effect on welfare from τ^* to T of being silent one additional instant.[16]

A different way to interpret the condition $\alpha > \frac{c}{T}$ from theorem 1 is that it provides a necessary and sufficient condition for having, in a neighborhood of $\tau = 0$, the marginal

[16]Formally marginal benefits are:

$$\frac{1}{p + P_\theta} + \frac{\alpha\tau}{p + P_\theta} - \frac{\alpha\tau(\alpha\tau + 2)}{p + P(T|\tau, \alpha)}$$

where $p + P(T|\tau, \alpha) = (p + P_\theta)[(1 + \alpha\tau)^3 + 3\alpha(T - \tau)]^{1/3}$.

benefits of delaying communication higher than its marginal costs. Since we argued that it is always optimal to communicate before T, the welfare function must exhibit decreasing marginal benefit of delaying communication, that is, welfare must be concave in τ, at least in a neighborhood of T. Given marginal benefits higher than marginal cost in a neighborhood of $\tau = 0$ then it will be optimal to delay communication, until decreasing marginal benefits kick in. Concavity of the welfare function in τ pins down τ^*.

It is not straightforward to analyze how parameters affect marginal costs and marginal benefits. They enter both terms in non-monotonic ways, obfuscating the comparative statics. However most analysis can be done by studying the effect of parameters on the jamming effect, α. The following proposition provides comparative statics results:

Proposition 5 (Comparative Statics)**.** *Let $c \in \mathbb{R}^{++}$ as in theorem (1), $\gamma \in \mathbb{R}^{++}$, $\gamma > 1$ and $\alpha \equiv \frac{p^2}{(p+P_\theta)^3} p_\varepsilon$. If $\tau^* > 0$ then*

i. *τ^* is monotonically increasing in T;*

ii. *there exists $\alpha' \equiv \gamma \frac{c}{T}$, such that τ^* is increasing in α for $\alpha < \alpha'$ and τ^* is decreasing in α for $\alpha > \alpha'$, moreover $\tau^* \to 0$ as $\alpha \to \infty$;*

iii. *there exists p'_ε such that τ^* is increasing in p_ε for $p_\varepsilon < p'_\varepsilon$, and decreasing in p_ε for $p_\varepsilon > p'_\varepsilon$;*

iv. *if $\frac{p_\varepsilon}{p} > \gamma \frac{c}{T}$ then there exists P'_θ such that τ^* is increasing in P_θ for $P_\theta < P'_\theta$ and is decreasing in P_θ for $P_\theta > P'_\theta$, otherwise τ^* is monotonically decreasing in P_θ;*

v. *let p^* be the unique maximizer of α over p. If $\alpha(p^*) > \gamma \frac{c}{T}$, then there exists p' and p'', $p' < p^* < p''$, such that:*

- *τ^* is increasing in p for $p < p'$ and for $p^* < p < p''$;*
- *τ^* in decreasing in p for $p' < p < p^*$ and for $p > p''$;*

otherwise:

- *τ^* is increasing in p for $p < p^*$;*
- *τ^* is decreasing in p for $p > p^*$*

Proof. In the appendix. □

Part *i.* of previous proposition is the only one whose intuition can be provided easily in terms of equalizing marginal benefits and marginal costs of not communicating. If the horizon over which the information flow between the agency and the financial institutions increases, T, then the benefits of delayed communications can be felt for a longer interval of time. This increases the marginal benefit of delayed communication, while marginal costs do not vary. The optimal revelation time increases, decreasing marginal benefit of delaying communication and reestablishing equality with marginal costs.

Figure 2 represents part *ii.* of proposition 5, which characterizes the hump-shaped behavior of the optimal delay time in communication. Why is τ^* non monotonic? There are

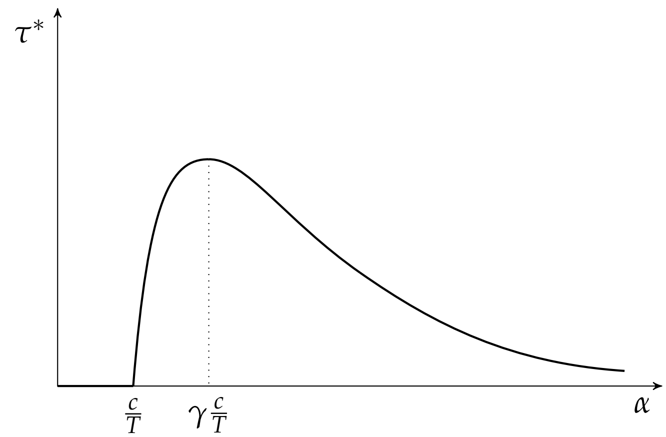

Figure 2: Optimal delayed plan as function of α

two connected effects pulling the optimal policy in opposite directions: the jamming effect and the precision of the signal the agency receives. The jamming effect, that is, the impairment in learning the agency experiences after communicating, is stronger the higher the precision of the signals the agency receives.[17] This is clear: when the agency has very precise information, communicating implies that financial institutions heavily rely on it, preventing future efficient production of information, and increasing the jamming effect. However, while the jamming effect pushes for a delay of public communication in order to efficiently learn, a more precise signal to the agency pushes towards an early revelation, since the agency is learning more. This explains the non monotonicity of τ^*: initially for low values of α, the jamming effects dominates. However, as the signal to the agency becomes more precise, this effect starts to dominates. The agency can learn faster and therefore it pushes for a shorter delay period.

For sake of clarity consider p_ε, the precision of the measurement error, since both the jamming effect and the precision of the signal to agency are monotone increasing transformation of p_ε. When p_ε is small (but such that it is optimal, for given other parameters' values, to delay communication), an increase in p_ε raises the jamming effect since the

[17] Remember from equation (16) that, when not communicating, the agency observes a signal whose precision is given by $\left(\frac{p}{p+P_\theta}\right)^2 p_\varepsilon$, and the jamming effect α, is given by $\frac{p^2}{(p+P_\theta)^3} p_\varepsilon$. Therefore the same change in parameters that makes the jamming effect increase, also increases the precision of the signal to the agency. This is not always true for p, as explained later.

agency has more precise information and public communication would make financial institutions heavily use it. However as the measurement error becomes smaller (p_ε increases), the agency is able to infer almost perfectly θ, therefore there is no need to wait long before communicating. That is why, as $p_\varepsilon \to \infty$, the optimal policy is to reveal as early as possible.[18]

The reaction of the optimal delay time τ^* with respect to changes in precision P_θ of the initial prior also depends on the interaction of the two effects, with an important change: an increase in P_θ now decreases both the jamming effect and the precision of the signal to the agency (since financial institutions are relying less on their private information). Therefore the hump-shaped behavior of τ^* obtains only if other parameters' values are such that the threshold that sets the limit of the jamming effect is achieved (threshold given by $\gamma\frac{c}{T}$). Otherwise an increase in the precision of public initial knowledge, by lowering the jamming effect, pushes towards early releases of information.

The comparative statics for p, the precision of private information, is quite involved: the two effects described above can change in opposite directions as p changes. While an increase in p always increases the precision of the signal the agency receives (since it is able to extract more information from the choices of the financial institutions), the effect of p on the jamming effect is non monotonic.[19] Intuitively, when p is sufficiently small, an increase in p increases the jamming effect since there is low general knowledge among financial institutions, and public information will be heavily used. On the other hand if p is sufficiently large, an increase in p decreases the jamming effect, since financial institutions already have sufficiently good private knowledge and will not rely too much on public information. This behavior, interacting with the two opposite jamming and increased learning effects, determines the oscillating behavior of τ^* as precision of private information changes.

[18]Morris and Shin (2005) discuss the contrasting effects of the increased power of observation for central banks as providing more information to the agents but also generating excessive coordination of agents' beliefs on public information, and so suppressing the channel through which dissenting agents can express their views. They call this the "paradox of transparency" (p. 19). The two effects described in Morris and Shin (2005) are here the increase in signal precision to the agency and the increase in the jamming effect as p_ε increases. The analysis in this paper on the one hand highlights a way to solve the paradox described by Morris and Shin, that is to delay communication. On the other hand, by providing exact comparative statics with respect to p_ε, it clarifies how the two effects balance each other in determining the optimal delay time.

[19]The non monotonicity depends on p and P_θ. If $p < 2P_\theta$, then $\frac{\partial \alpha}{\partial p} > 0$.

4 Conclusions and further directions for research

This works builds a model of the information flow between financial institutions and a public agency. The agency learns about a fundamental of the economy only by observing the actions of the financial institutions and has to decide upon a plan of public communication. Given the information structure, we show that there is a negative feedback between public communication today and tomorrow, such that, if the agency decides to publicly communicate its beliefs, this implies a lower informational value of future public communication. The paper characterizes public communication plans that maximize ex-ante aggregate welfare and shows that, under some conditions, optimal communication plan involves delay, that is, an initial period in which public communication is absent followed by a period of public communication. Delayed communication can be optimal especially when financial institutions have scarce knowledge of the fundamentals of the economy. When it is optimal to delay communication two opposite forces determine the delay time: excessive reliance on public information, the jamming effect, and increased learning of the agency. In addition we show that there are no other public communication plans that achieve higher welfare than the simple bang-bang communication plan.

The general implications of this research are twofolds: on the one hand it describes how, when information is endogenously created, transparency and public communication can have indirect effects on welfare by the mechanism of information production itself, something that was not pointed out in the literature before. On the other hand it highlights the fact that timing of public communication is fundamental, even in situations when information evolves continuously and there are no shocks to the system. Governmental agencies involved in public communication should take into account these two features of the communication decision when thinking about disclosure of information.

The direct continuation of the analysis in this paper involves introducing markets. This can be done by borrowing from the finance literature on noise traders. The price will effectively constitute a public signal always available to the financial institutions. However public communication by the agency will not become irrelevant. Since its origin lies in a different mechanism than price formation it will offer information orthogonal to prices. In addition if precision of private knowledge is low and prices do not constitute a sufficiently precise signal because of large amount of noise trading, then knowledge of the financial institutions will be low and in this case the jamming effect can be quite strong. This will imply that the basic trade-off between present and future communication will always be present. The introduction of markets has the implication that now the agency will have to choose whether to let financial institutions learn from markets only or to contribute to the information mechanism of prices.

This work does not allow for the richest action space for the agency. The most general setting would be to allow the agency to add noise to its public signal, and decide on the precision of this noise in each period. This work restricted attention to communication plans that assume precision of noise of either 0, when the agency is not communicating, or infinity, when the agency is communicating exactly its mean beliefs. By introducing the choice of precision of each signal, we can characterize the optimal *path* of precisions of the public communication plan. This extension, however, generates extensive across time correlation of the public signal, which prevents closed form proofs of propositions and makes necessary the use of computational techniques. Miccoli (2010) studies this problem of choosing precision of the public signal in each period in a easier setting, but with the unknown parameter evolving over time as a random walk, and shows that optimal communication plans involve *garbling*, that is, adding a finite amount of noise to the public signal. This result is able to provide a justification for *opaque* public communication. For instance, FOMC statements are often seen as deliberately ambiguous about the state of the economy.

On a more general point, the main message from this paper is that, when public communication is about *fundamentals* of the economy, then transparency might not be always optimal. Public communication about fundamentals is only one of the two types of communication public agencies are involved with. The other type is public communication about public agencies' decisions. Here transparency is needed in order to create correct expectations of their behavior, and this is optimal as the literature stemming from the Barro-Gordon model analyzes. For instance, Gosselin, Lotz and Wyplosz (2008) build a model in which public communication influence agents' expectations formations about behavior of the Central Bank, and as such is welfare enhancing.

However future policy decisions are based on estimates of fundamentals and this implies that there is an connection between the two types of communication of public agencies. Consider the rate of interest policy of the Fed: any communication about future FOMC decisions helps the investment decisions of the agents in the economy. But choosing a future rate of interest cannot prescind from the information the Fed has about the business cycle, hence about fundamentals of the economy. Communication about future policy decisions therefore also reveals the Fed beliefs about deep parameters, and, by influencing the actions of the agents in the economy, also affects the beliefs of the Fed about the state of the world. Studying how communication about fundamentals affects communication about future policy decisions and viceversa, and what are the welfare effects of each of them, is the next step in characterizing optimal public communication plans.

A Proofs not in main body

A.1 Proof of proposition 1, Aggregate ex-ante Welfare

Proof. The aggregate ex-ante period t contribution to welfare is given by

$$\mathbb{E}_0 \left[x_{i,t}(\theta - R) - \frac{\lambda R}{2} x_{i,t}^2 \right] \implies \frac{1}{2\lambda R} \mathbb{E}_0 \left[(\mathbb{E}_{i,t}[\theta] - R)^2 \right]$$

where we made use of optimal change in positions $x_{i,t} = \frac{\mathbb{E}_{i,t}[\theta]-R}{\lambda R}$ and the law of iterated expectation. Expanding the square:

$$\frac{1}{2\lambda R} \mathbb{E}_0 \left[(\mathbb{E}_{i,t}[\theta] - R)^2 \right] = \frac{1}{2\lambda R} \left(\mathbb{E}_0 \left[\mathbb{E}_{i,t}[\theta]^2 \right] - 2R\mathbb{E}_0 \left[\mathbb{E}_{i,t}[\theta] \right] + R^2 \right).$$

By iterated expectation $\mathbb{E}_0 \left[\mathbb{E}_{i,t}[\theta] \right] = \mathbb{E}_0[\theta] = \bar{\theta}$, and

$$\mathbb{E}_0 \left[\mathbb{E}_{i,t}[\theta]^2 \right] = \mathbb{E}_0 \left[\mathbb{E}_{i,t}[\theta^2] - \mathbb{E}_{i,t}[\theta^2] + \mathbb{E}_{i,t}[\theta]^2 \right] = \mathbb{E}_0[\theta^2] - \mathbb{V}ar_t[\theta] = \mathbb{E}_0[\theta^2] - \frac{1}{p + P_t}.$$

The first expectation can be computed using results for the non-central chi-squared distribution, $\mathbb{E}_0[\theta^2] = \frac{1}{P_\theta} + \bar{\theta}^2$. By repeating these steps for all periods t and summing them up, we obtain the expression for ex-ante aggregate welfare:

$$B_{-1}R + D_{-1}\bar{\theta} + \beta(T+1) \left[\frac{1}{P_\theta} + (\bar{\theta} - R)^2 \right] - \beta \sum_{t=0}^{T} \frac{1}{p + P_t}$$

where $\beta \equiv \frac{1}{2\lambda R}$ and B_{-1} and D_{-1} represents respectively the initial aggregate endowment of risky and riskless technology. By letting $\lambda = R = 1$, the result is obtained. □

A.2 Proof of lemma 1, Evolution of Precision with Delayed Communication

Proof. i. We will first prove the statement for $t = \tau$. By contradiction suppose that $P(\tau) + p \leq P'(\tau) + p$, then

$$\begin{aligned} P(\tau) + p \leq P'(\tau) + p &\implies (1 + \alpha\tau) \leq [1 + 3\alpha\tau]^{1/3} \\ &\implies (1 + \alpha\tau)^3 \leq [1 + 3\alpha\tau] \qquad \text{(by the cubic monotone transformation)} \\ &\implies 3\alpha^2\tau^2 + \alpha^3\tau^3 \leq 0 \qquad \text{(by expanding cube and cancelling terms)} \end{aligned}$$

which cannot be true since $\tau > 0$ and $\alpha > 0$. Hence $P(\tau) + p > P'(\tau) + p$.

ii. Now let's consider $t > \tau$. By contradiction suppose that there exists some $\bar{t} > \tau$ such that

$P(\bar{t}) + p \leq P'(\bar{t}) + p$. Then by i. and continuity there must exist $\tilde{t} > \tau$ such that $P(\tau) + p = P'(\tau) + p$. By solving the expression we obtain $3\alpha^2\tau^2 + \alpha^3\tau^3 = 0$, which again cannot be true since $\tau > 0$ and $\alpha > 0$.

□

A.3 Proof of theorem 1, Delayed Optimal Communication

Proof. The proof involves solving the maximization problem of the agency under the delayed communication plan. There is no assurance of concavity of the problem, hence we need to study the sign of the first derivative in order to ascertain whether solutions to first order conditions are maximizers or minimizers of the problem. Since we will provide conditions under which maximizers satisfy first and second order conditions, the conditions found are necessary and sufficient.

Plugging the solution to the ODE in the expression for ex-ante welfare, and solving the integral, we obtain:

$$\begin{aligned} W(\tau, T) &= \kappa - \int_0^\tau \frac{1}{p + P_\theta} dt - \int_\tau^T \frac{\left[(1+\alpha\tau)^3 + 3\alpha(t-\tau)\right]^{-1/3}}{p + P_\theta} dt \\ &= \kappa - \frac{\tau}{p+P_\theta} - \frac{1}{2(p+P_\theta)\alpha}\left\{\left[(1+\alpha\tau)^3 + 3\alpha(T-\tau)\right]^{2/3} - (1+\alpha\tau)^2\right\} \\ &\propto -\tau\alpha - \frac{1}{2}\left[(1+\alpha\tau)^3 + 3\alpha(T-\tau)\right]^{2/3} + \frac{1}{2}(1+\alpha\tau)^2 \end{aligned}$$

where terms which affect only the level of the function have been excluded. Let $x \equiv 1 + \alpha\tau$ and $W(\tau, T) \equiv \omega(x)$. Since $\tau \in [0, T]$, then $x \in [1, 1 + \alpha T]$. We want to solve:

$$\max_{x \in [1, 1+\alpha T]} \omega(x) = \max_{x \in [1, 1+\alpha T]} \left\{-x + 1 - \frac{1}{2}\left[x^3 + 3\alpha T - 3(x-1)\right]^{2/3} + \frac{1}{2}x^2\right\}$$

One can check that $\omega(x)$ is not concave in x for all parameters values, therefore we will rely on studying the sign of the first derivative for finding maxima. Taking the first order condition and setting it equal to 0:

$$\begin{aligned} \frac{\partial \omega(x)}{\partial x} &= -1 - \left[x^3 + 3\alpha T - 3(x-1)\right]^{-1/3}(x^2 - 1) + x = 0 \\ &\implies (x-1)\left[1 - (x+1)\left[x^3 + 3\alpha T - 3(x-1)\right]^{-1/3}\right] = 0 \end{aligned}$$

The equation has two solutions, the first one being $x = 1$ which is the corner solution $\tau = 0$. The second one is obtained by solving:

$$1 - (x+1)\left[x^3 + 3\alpha T - 3(x-1)\right]^{-1/3} = 0 \quad \implies \quad 3x^2 + 6x - 3\alpha T - 2 = 0$$

This is a quadratic equation, whose solutions are given by $x_{1,2} = -1 \pm \sqrt{5/3 + \alpha T}$. The solution $x_2 = -1 - \sqrt{5/3 + \alpha T}$ always falls out of the admissible range of x, hence we can discard it. The other is inside the admissible range of x if and only if $(-1 + \sqrt{5/3 + \alpha T}) > 1$, which is true if and only if $\alpha > \frac{7}{3T}$. Note that this solution is always lesser than the upper bound of the range of x since $(-1 + \sqrt{5/3 + \alpha T}) < 1 + \alpha T$ for all $\alpha, T > 0$.

In order to choose which solutions are maxima or minima we will study the sign of the first derivative of $\omega(x)$. At the extrema of the admissible range of x we have that $\left[\frac{\partial \omega(x)}{\partial x}\right]_{x=1} = 0$, and $\left[\frac{\partial \omega(x)}{\partial x}\right]_{x=1+\alpha T} = -\frac{\alpha T}{1+\alpha T} < 0$, for all $\alpha, T > 0$. There are two cases two consider, one when $\alpha > \frac{7}{3T}$ and there are two critical points, and the other one when $\alpha \leq \frac{7}{3T}$ and there is only one critical point.

- *Case 1:* $\alpha > \frac{7}{3T}$

  By the analysis of the solution of the first order condition equalized to 0, we know that if $\alpha > \frac{7}{3T}$, $\frac{\partial \omega(x)}{\partial x} = 0$ only once for $x \in (1, 1 + \alpha T]$, when $x^* = -1 + \sqrt{5/3 + \alpha T}$. Since $\left[\frac{\partial \omega(x)}{\partial x}\right]_{x=1+\alpha T} < 0$ then by continuity $\frac{\partial \omega(x)}{\partial x} < 0$ for $x \in (x^*, 1 + \alpha T]$.

  We want to understand the sign of $\frac{\partial \omega(x)}{\partial x}$ in a neighborhood of $x = 1$. Let $\frac{\partial \omega(x)}{\partial x} = f(x)g(x)$, where $f(x) = x - 1$ and $g(x) = 1 - (x+1)\left[x^3 + 3\alpha T - 3(x-1)\right]^{-1/3}$. $g(1) = 1 - 2(1 + 3\alpha T)^{-1/3} > 0$ for $\alpha > \frac{7}{3T}$. Therefore, by continuity, $\frac{\partial \omega(x)}{\partial x} > 0$ for x in a neighborhood of 1. But since $\frac{\partial \omega(x)}{\partial x} = 0$ only at x^*, then $\frac{\partial \omega(x)}{\partial x} > 0$ for $x \in (1, x^*)$.

  Therefore when $\alpha > \frac{7}{3T}$, $x = 1$ is not a maximizer, and $x^* = -1 + \sqrt{5/3 + \alpha T}$ is the unique global maximizer of $\omega(x)$.

- *Case 2:* $\alpha \leq \frac{7}{3T}$

  When $\alpha \leq \frac{7}{3T}$, since the first order condition will be equal to 0 only at $x = 1$, and will be negative at the upper bound of x, by continuity $\frac{\partial \omega(x)}{\partial x} < 0$ for $x \in (1, 1 + \alpha T]$, therefore $x^* = 1$ is the unique global maximizer.

Remembering that $\tau = \frac{x-1}{\alpha}$, and letting $c \equiv \frac{7}{3}$, the result is obtained. $\square$

A.4 Proof of theorem 2, Optimal communication in general action space

Proof. The proof is an algorithm for finding the solutions of the maximization problem. Given that for each solution first and second order conditions holds, it will define necessary and sufficient conditions for optimality.

There are many building blocks to the proof, however it is based on three simple observations: i) the amount of precision of public knowledge at any point in time is the only relevant information the agency needs for choosing the next communication spell; ii) the problem can be decomposed into choosing optimally the last element of the sequence given everything else; iii) choosing the last element of the sequence implies solving a problem that is isomorphic to the one solved in proposition (1).

In order to prove the statement the following three lemmas provide the basic building blocks. The first lemma is the special case for $n = 1$ of the statement we want to prove.

Lemma 2. *Consider a communication plan as the following:*

$$\Theta_t = \begin{cases} \emptyset & 0 \leq t < \tau_1 \\ \mathbb{E}_{A,t}[\theta] & \tau_1 \leq t < \tau_2 \\ \emptyset & \tau_2 \leq t < \tau_3 \\ \mathbb{E}_{A,t}[\theta] & \tau_3 \leq t \leq T \end{cases} \tag{24}$$

with $0 \leq \tau_1 \leq \tau_2 \leq \tau_3 < T$*. Let* $\tau^* = \arg\max_{\tau \in [0,T]} W(\tau)$*, as in theorem (1), and let* $\{\tau_1^*, \tau_2^*, \tau_3^*\} = \arg\max_{\{\tau_1,\tau_2,\tau_3\}} W(\tau_1, \tau_2, \tau_3)$*. Then the optimal policy* $\{\tau_1^*, \tau_2^*, \tau_3^*\}$ *as specified in communication plan (24) will replicate the communication policy under a delayed plan in theorem (1), that is, either* $\tau_2^* - \tau_1^* = 0$ *and* $\tau_3^* = \tau^*$*, or* $\tau_1^* = \tau^*$ *and* $\tau_3^* - \tau_2^* = 0$*.*

Proof. The precision of public knowledge implied by using a communication plan as in (24) for arbitrary $\tau_1 \leq \tau_2 \leq \tau_3$ is found by using the the continuous time version of evolution of public knowledge (19) together with the boundaries conditions. The first boundary condition is given by the amount of public knowledge at τ_1, $P(\tau_1) = P_\theta + \tau_1 \left(\frac{p}{p+P_\theta}\right)^2 p_\varepsilon$. The second boundary condition is defined by the amount of public knowledge in τ_3. When the agency starts communicating in τ_3, it will communicate all the signals observed. However only the ones received during $(\tau_3 - \tau_2)$ constitute new public information. Hence the boundary condition in τ_3 is $P(\tau_3) = P(\tau_2) + (\tau_3 - \tau_2)\left(\frac{p}{p+P(\tau_2)}\right)^2 p_\varepsilon$. The precision of public knowledge implied by such communication plan is therefore given by:

$$P(t) = \begin{cases} P_\theta & 0 \leq t < \tau_1 \\ -p + (P_\theta + p)\left[(1+\alpha\tau_1)^3 + 3\alpha(t-\tau_1)\right]^{1/3} & \tau_1 \leq t < \tau_2 \\ -p + (P_\theta + p)\left[(1+\alpha\tau_1)^3 + 3\alpha(\tau_2-\tau_1)\right]^{1/3} & \tau_2 \leq t < \tau_3 \\ -p + (P(\tau_2) + p)\left[(1 + \frac{\alpha(\tau_3-\tau_2)}{(1+\alpha\tau_1)^3+3\alpha(\tau_2-\tau_1)})^3 + \frac{3\alpha(t-\tau_3)}{(1+\alpha\tau_1)^3+3\alpha(\tau_2-\tau_1)}\right]^{1/3} & \tau_3 \leq t < T \end{cases} \tag{25}$$

where $\alpha \equiv \frac{p^2 p_\varepsilon}{(P_\theta+p)^3}$.

We are going to analyze sequentially optimal choices, i.e. analyze what is the optimal value of τ_3^* for given arbitrary τ_1, τ_2, then the optimal τ_2 for given τ_1 and τ_3^*, finally the optimal τ_1 for given τ_2^*, τ_3^*.

The attainable welfare as function of (τ_1, τ_2, τ_3) is:

$$
\begin{aligned}
W(\tau_1,\tau_2,\tau_3) = & \int_{t=0}^{T} \left[(\bar{\theta}-1)^2 + \frac{1}{P_\theta}\right] dt - \int_{t=0}^{\tau_1} \frac{1}{p+P_\theta} dt - \int_{t=\tau_1}^{\tau_2} \frac{1}{p+P(t)} dt - \int_{t=\tau_2}^{\tau_3} \frac{1}{p+P(\tau_2)} dt - \int_{t=\tau_3}^{T} \frac{1}{p+P(t)} dt \\
\propto & \frac{1}{2\alpha(P_\theta+p)} \left\{ -2\alpha\tau_1 + (1+\alpha\tau_1)^2 - \left[(1+\alpha\tau_1)^3 + 3\alpha(\tau_2-\tau_1)\right]^{2/3} - \frac{2\alpha(\tau_3-\tau_2)}{\left[(1+\alpha\tau_1)^3+3\alpha(\tau_2-\tau_1)\right]^{1/3}} + \right. \\
& - \left[(1+\alpha\tau_1)^3 + 3\alpha(\tau_2-\tau_1)\right]^{2/3} \left[\left(1+\frac{\alpha(\tau_3-\tau_2)}{(1+\alpha\tau_1)^3+3\alpha(\tau_2-\tau_1)}\right)^3 + \frac{3\alpha(T-\tau_3)}{(1+\alpha\tau_1)^3+3\alpha(\tau_2-\tau_1)}\right]^{2/3} + \\
& \left. + \left[(1+\alpha\tau_1)^3 + 3\alpha(\tau_2-\tau_1)\right]^{2/3} \left(1+\frac{\alpha(\tau_3-\tau_2)}{(1+\alpha\tau_1)^3+3\alpha(\tau_2-\tau_1)}\right)^2 \right\}
\end{aligned}
$$

where in the second line terms that do not depend on the optimization problem have been omitted.

Define $x \equiv (1+\alpha\tau_1)^3 + 3\alpha(\tau_2 - \tau_1)$ and $y \equiv (\tau_3 - \tau_2)$. Given that we are considering a given τ_1 for the moment, x defines a transformation on the choice variable τ_2 and y a transformation on the choice variable τ_3. The restrictions on the parameter space of (τ_2, τ_3), $\tau_1 \leq \tau_2 \leq \tau_3 \leq T$, imply that $x \in [(1+\alpha\tau_1)^3, (1+\alpha\tau_1)^3 + 3\alpha(T-\tau_1)]$, $y \in [0, T-\tau_1]$ and $x + 3\alpha y \leq 3\alpha T - 3\alpha\tau_1 + (1+\alpha\tau_1)^3$. Substituting in the welfare function (and omitting constant terms), we obtain:

$$
\begin{aligned}
W(\tau_1, x, y) = & -2\alpha\tau_1 + (1+\alpha\tau_1)^2 - x^{2/3} - \frac{2\alpha y}{x^{1/3}} + \\
& - x^{2/3} \left\{ \left[\left(1+\frac{\alpha y}{x}\right)^3 + \frac{3\alpha T}{x} - \frac{3\alpha y + x + 3\alpha\tau_1 - (1+\alpha\tau_1)^3}{x}\right]^{2/3} - \left(1+\frac{\alpha y}{x}\right)^2 \right\}
\end{aligned}
\tag{26}
$$

Taking the derivative with respect to y of $W(\tau_1, x, y)$ yields:

$$
\begin{aligned}
\frac{\partial W}{\partial y} = 2\alpha & \left\{ x^{-1/3}\left(\frac{x+\alpha y}{x} - 1\right) + \right. \\
& \left. - \left[\frac{(x+\alpha y)^3}{x^2} + 3\alpha(T-\tau_1) - 3\alpha y - x + (1+\alpha\tau_1)^3\right]^{-1/3} \left[\left(\frac{x+\alpha y}{x}\right)^2 - 1\right] \right\}
\end{aligned}
$$

$\frac{\partial W}{\partial y} = 0$ implies

$$
\left(\frac{x+\alpha y}{x} - 1\right) = x^{1/3} [\ldots]^{-1/3} \left(\frac{x+\alpha y}{x} + 1\right)\left(\frac{x+\alpha y}{x} - 1\right) \tag{27}
$$

The first solution obtains when $\left(\frac{x+\alpha y}{x} - 1\right) = 0$, which is $y = 0$. The other solutions are found by solving:

$$
x^{-1/3} [\ldots]^{1/3} = \left(\frac{x+\alpha y}{x} + 1\right)
$$

After some algebraic manipulation you can obtain a quadratic equation in y given by:

$$y^2 + 4\frac{x}{\alpha}y + \frac{8}{3}\left(\frac{x}{\alpha}\right)^2 - \frac{x}{\alpha}\left[(T-\tau_1) + \frac{(1+\alpha\tau_1)^3}{3\alpha}\right] = 0$$

whose solutions are $y = \frac{-6x\pm\sqrt{3}\sqrt{4x^2+x[(1+\alpha\tau_1)^3+3\alpha(T-\tau_1)]}}{3\alpha}$. Summarizing, the first order condition $\frac{\partial W}{\partial y} = 0$ has three solutions with respect to y:

$$\begin{aligned} y_1 &= \frac{-6x - \sqrt{3}\sqrt{4x^2 + x[(1+\alpha\tau_1)^3 + 3\alpha(T-\tau_1)]}}{3\alpha}, \\ y_2 &= \frac{-6x + \sqrt{3}\sqrt{4x^2 + x[(1+\alpha\tau_1)^3 + 3\alpha(T-\tau_1)]}}{3\alpha}, \\ y_3 &= 0 \end{aligned}$$

The first solution, y_1, is not admissible since it falls outside the range of y. The third solution, $y_3 = 0$ is always admissible.

It can be checked that $y_2 < T - \tau_1$ for all $x \in [(1+\alpha\tau_1)^3, (1+\alpha\tau_1)^3 + 3\alpha(T-\tau_1)]$,$\tau_1 \geq 0$, for all $\alpha > 0, T > 0$. On the other hand $y_2 \geq 0$ if and only if $-6x + \sqrt{3}\sqrt{4x^2 + x[(1+\alpha\tau_1)^3 + 3\alpha(T-\tau_1)]} \geq 0$, that is, if and only if $x \leq \frac{(1+\alpha\tau_1)^3+3\alpha(T-\tau_1)}{8}$. Given the restriction on the parameter space of x there are several possible cases:

- Case 1: $\alpha < \frac{7}{3T}$
  $\forall\tau_1 \geq 0, \frac{(1+\alpha\tau_1)^3+3\alpha(T-\tau_1)}{8} \notin [(1+3\alpha\tau_1)^3, (1+\alpha\tau_1)^3 + 3\alpha(T-\tau_1)] \implies y_2 \notin [0, T-\tau_1]$
- Case 2: $\alpha = \frac{7}{3T}$
  - Sub-case i: $\tau_1 = 0$
    $\frac{(1+\alpha\tau_1)^3+3\alpha(T-\tau_1)}{8} = (1+\alpha\tau_1)^3 \implies y_2 = 0$
  - Sub-case ii: $\tau_1 > 0$
    $\frac{(1+\alpha\tau_1)^3+3\alpha(T-\tau_1)}{8} \notin [(1+3\alpha\tau_1)^3, (1+\alpha\tau_1)^3 + 3\alpha(T-\tau_1)] \implies y_2 \notin [0, T-\tau_1]$
- Case 3: $\alpha > \frac{7}{3T} \implies \exists\tilde{\tau}_1 > 0 : \frac{(1+\alpha\tilde{\tau}_1)^3+3\alpha(T-\tilde{\tau}_1)}{8} = (1+\alpha\tilde{\tau}_1)^3$
  - Sub-case i: for $\tau_1 < \tilde{\tau}_1 \implies$
    $\frac{(1+\alpha\tau_1)^3+3\alpha(T-\tau_1)}{8} \in [(1+3\alpha\tau_1)^3, (1+\alpha\tau_1)^3 + 3\alpha(T-\tau_1)] \implies$
    $$\begin{cases} y_2 \in (0, T-\tau_1] & (1+\alpha\tau_1)^3 \leq x < \frac{(1+\alpha\tau_1)^3+3\alpha(T-\tau_1)}{8} < 3\alpha(T-\tau_1) + (1+\alpha\tau_1)^3 \\ y_2 = 0 & (1+\alpha\tau_1)^3 < x = \frac{(1+\alpha\tau_1)^3+3\alpha(T-\tau_1)}{8} < 3\alpha(T-\tau_1) + (1+\alpha\tau_1)^3 \\ y_2 \notin [0, T-\tau_1] & (1+\alpha\tau_1)^3 < \frac{(1+\alpha\tau_1)^3+3\alpha(T-\tau_1)}{8} < x \leq 3\alpha(T-\tau_1) + (1+\alpha\tau_1)^3 \end{cases}$$
  - Sub-case ii: for $\tau_1 > \tilde{\tau}_1 \implies$
    $\frac{(1+\alpha\tau_1)^3+3\alpha(T-\tau_1)}{8} \notin [(1+3\alpha\tau_1)^3, (1+\alpha\tau_1)^3 + 3\alpha(T-\tau_1)] \implies y_2 \notin [0, T-\tau_1]$
  - Sub-case iii: for $\tau_1 = \tilde{\tau}_1 \implies$

$$\begin{cases} y_2 = 0 & (1+\alpha\tau_1)^3 = \frac{(1+\alpha\tau_1)^3+3\alpha(T-\tau_1)}{8} = x < 3\alpha(T-\tau_1)+(1+\alpha\tau_1)^3 \\ y_2 \notin [0, T-\tau_1] & (1+\alpha\tau_1)^3 = \frac{(1+\alpha\tau_1)^3+3\alpha(T-\tau_1)}{8} < x \leq 3\alpha(T-\tau_1)+(1+\alpha\tau_1)^3 \end{cases}$$

What the different cases imply for the optimal solution is explained below.

- Case 1 and 2: $\alpha \leq \frac{7}{3T}$

  The only admissible solution is $y = 0$, either because $y_2 < 0$ or because $y_2 = y_1 = 0$. It can be checked that in these parametrization $\frac{\partial W}{\partial y} < 0$ for all x, y, τ_1 in the admissible range. In fact, notice from equation (27) that the sign of $\frac{\partial W}{\partial y}$ depends on

  $$x^{-1/3} - \left[\frac{(x+\alpha y)^3}{x^2} + 3\alpha(T-\tau_1) - 3\alpha y - x + (1+\alpha\tau_1)^3\right]^{-1/3}\left(\frac{x+\alpha y}{x}+1\right).$$

  In particular $\frac{\partial W}{\partial y} < 0 \iff x^{-1/3}[\ldots]^{1/3} - \left(\frac{x+\alpha y}{x}+1\right) < 0$. By manipulating this expression along steps done above we obtain

  $$\frac{\partial W}{\partial y} < 0 \iff y^2 + 4\frac{x}{\alpha}y + \frac{8}{3}\left(\frac{x}{\alpha}\right)^2 - \frac{x}{\alpha}\left[(T-\tau_1) + \frac{(1+\alpha\tau_1)^3}{3\alpha}\right] > 0$$

  For $\alpha \leq \frac{7}{3T}$, y_1 and y_2 (roots of the quadratic expression) are both non-positive, hence the quadratic expression will be always positive for x, y, τ_1 in the admissible range. Therefore $\frac{\partial W}{\partial y} < 0$ and $y^* = 0$ is a global maximum. But this implies that $\tau_2 = \tau_3^*$, hence in the optimal solution we are foregoing the possibility to be silent between τ_2 and τ_3.

- Case 3: $\alpha > \frac{7}{3T}$
  - when $\tau_1 < \tilde{\tau}_1$, $(1+\alpha\tau_1)^3 \leq x < \frac{(1+\alpha\tau_1)^3+3\alpha(T-\tau_1)}{8} < 3\alpha(T-\tau_1)+(1+\alpha\tau_1)^3$, then $y_2 > 0$. There are therefore two solutions. By studying the sign of the first order condition along steps to the ones done above we can show that $\frac{\partial W}{\partial y} > 0$ for $0 < y < y_2$ (with equality at the extrema), while $\frac{\partial W}{\partial y} < 0$ for $y > y_2$. Hence $y^* = y_2 > 0$ is a global maximum and $y = 0$ is a local minimum.
  - in all other cases $y_2 < 0$, the only solution is $y^* = 0$ which can be shown to be a maximum.

Now consider:

$$W(y^* = 0, x, \tau_1) = -2\alpha\tau_1 + (1+\alpha\tau_1)^2 - [(1+\alpha\tau_1)^3 + 3\alpha(T-\tau_1)]^{2/3}$$

Inserting the optimal solution $y^* = 0$ in the welfare function, the x disappears (clearly now the agency is communicating between τ_1 and T, hence the choice of τ_2 is irrelevant), and all we are left with is to find the optimal τ_1. But $W(y^* = 0, x, \tau_1)$ defines the same problems as theorem (1): the optimal solution will replicate the results of that theorem. Therefore $\tau_3^* - \tau_2^* = 0$ and $\tau_1^* = \tau^*$.

On the other hand, when $y^* > 0$,

$$W(y^* > 0, x, \tau_1) = 1 + (\alpha\tau_1)^2 + 4x^{2/3} - \frac{4}{\sqrt{3}} x^{1/6} \sqrt{(1+\alpha\tau_1)^3 + 3\alpha(T-\tau_1) + 4x}$$

By taking the derivative with respect to x:

$$\begin{aligned}\frac{\partial W(y^*(x), x)}{\partial x} = \frac{8}{3} x^{-1/3} - \frac{4}{6\sqrt{3}} x^{-5/6} \left[(1+\alpha\tau_1)^3 + 3\alpha(T-\tau_1) + 4x\right]^{1/2} + \\ - \frac{8}{\sqrt{3}} x^{1/6} \left[(1+\alpha\tau_1)^3 + 3\alpha(T-\tau_1) + 4x\right]^{-1/2}\end{aligned}$$

With some algebraic manipulation we obtain that

$$\frac{\partial W(y^*(x), x)}{\partial x} < 0 \iff \left(\frac{1}{6} \frac{(1+\alpha\tau_1)^3 + 3\alpha(T-\tau_1)}{x} - \frac{4}{3}\right)^2 > 0$$

Given that this expression is a square, it is always positive and so $\frac{\partial W(y^*(x),x)}{\partial x} < 0$ for x, τ_1 in the relevant range. Therefore the global maximum will be achieved at the corner $x^* = (1+\alpha\tau_1)^3$. This implies that $\tau_2^* = \tau_1$: there is no communication period $[\tau_1, \tau_2]$. The choice of τ_1 is hence irrelevant, because the agency will only choose when to start to communicate, τ_3. The problem becomes then similar to the one studied in theorem (1). In fact, $y^*(x = 1, \tau_1 = 0) = \frac{-2+\sqrt{5/3+\alpha T}}{\alpha}$, which is the optimal solution found before in theorem (1) when $\alpha > \frac{7}{3T}$.

□

The second building block of the proof is the following lemma:

Lemma 3. *Let $n \geq 1$ and consider an arbitrary sequence $\tau_1 \leq \tau_2 \leq \cdots \leq \tau_{2n+1}$ defining a general communication plan as in section 3.3. If for some $j = 1, \ldots, 2n+1$, $\frac{p^2}{(p+P(\tau_j))^3} p_\varepsilon > \frac{c}{T-\tau_j}$, $c \in \mathbb{R}^{++}$, then:*

- *It will also hold for all $k < j$;*
- *it will always be that $\frac{p^2}{(p+P_\theta)^3} p_\varepsilon > \frac{c}{T}$.*

Proof. $P(\tau_j)$ is weakly increasing in its argument (public knowledge can only weakly increase over time), therefore the left hand side of the inequality weakly increases for $\tau_k \leq \tau_1$, $k < j$. Also, for all $j \geq 1$, $P(\tau_j) \geq P_\theta$. On the other hand the right hand side of the inequality weakly decreases for $\tau_k \leq \tau_j$, $k < j$, hence the statement. □

The following is the last building block.

Lemma 4. *For all $n \geq 1$, if $\frac{p^2}{(p+P(\tau_{2n}))^3} p_\varepsilon > \frac{c}{T-\tau_{2n}}$, then for all $\tau_j \in C_{2n+1}$, $j = 1, \ldots, 2n+1$, $W(\tau^*) \geq W(C_{2n+1})$, with equality if and only if, for any $0 \leq k \leq n$, $\tau_{2k+1} = \tau^*$, $\tau_{2(j+1)} - \tau_{2j+1} = 0$ for all $j < k$ and $\tau_{2j+1} - \tau_{2j} = 0$ for all $j > k$.*

Proof. Step 1

If $n = 1$ then the statement is true by lemma (2). Therefore fix some $n > 1$ and an arbitrary sequence $\tau_1 \leq \tau_2 \leq \cdots \leq \tau_{2n+1}$. Consider the choice of the last element only, given all previous cut-points, that is solving:

$$\max_{\tau_{2n+1} \in [\tau_{2n}, T]} W(C_{2n}, \tau_{2n+1})$$

where $C_{2n} = C_{2n+1} \setminus \tau_{2n+1}$. The problem is formally isomorphic to the one studied in theorem (1): the choice of the length of the silence period $[\tau_{2n}, \tau_{2n+1})$, for fixed τ_{2n}, given that in $[\tau_{2n+1}, T]$ the agency is communicating. The important elements will be τ_{2n}, the lower bound, $P(\tau_{2n})$, the amount of public knowledge summarizing past communication decisions, and $T - \tau_{2n}$, the time horizon. But then, using proposition (1), a particular version of the threshold result will hold, that is if $\alpha(\tau_{2n}) \equiv \frac{p^2}{(p+P(\tau_{2n}))^3} p_\varepsilon > \frac{c}{T-\tau_{2n}}$ then the maximizer will be strictly greater than the lower bound, that is $\tau^*_{2n+1}(C_{2n}) > \tau_{2n}$, otherwise $\tau^*_{2n+1}(C_{2n}) = \tau_{2n}$. The only difference between this threshold rule and the one of proposition (1) will be in the amount of public knowledge at beginning of the considered interval, $P(\tau_{2n})$, and in the length of the interval, $T - \tau_{2n}$. Since by assumption $\frac{p^2}{(p+P(\tau_{2n}))^3} p_\varepsilon > \frac{c}{T-\tau_{2n}}$ holds, then $\tau^*_{2n+1} > \tau_{2n}$

Step 2

Now consider the problem of

$$\max_{\{\tau_{2n}, \tau^*_{2n+1}(C_{2n})\} \in [\tau_{2(n-1)+1}, T]} W(C_{2(n-1)+1}, \tau_{2n}, \tau^*_{2n+1}(C_{2n}))$$

that is, choosing the cut-points τ_{2n} and τ^*_{2n+1}, given that the agency is communicating in $[\tau_{2(n-1)+1}, \tau_{2n})$, is not communicating in $[\tau_{2n}, \tau^*_{2n+1})$, and is communicating again in $[\tau^*_{2n+1}, T]$. By lemma (3) we know that $\frac{p^2}{(p+P(\tau_{2(n-1)+1}))^3} p_\varepsilon > \frac{c}{T-\tau_{2(n-1)+1}}$. But then the problem is isomorphic to the one analyzed in lemma (2) after having set $\tau_1 = 0$. Therefore by lemma (2), $\tau^*_{2n}(C_{2(n-1)+1}) = \tau_{2(n-1)+1}$ and $\tau^*_{2n+1}(C_{2n}) > \tau_{2(n-1)+1}$.

Step 3

Given that optimally $\tau^*_{2n}(C_{2(n-1)+1}) = \tau_{2(n-1)+1}$ we can ignore from now on the choice of τ^*_{2n}. Now consider solving:

$$\max_{\{\tau_{2(n-1)+1}, \tau^*_{2n+1}(C_{2(n-1)+1}\} \in [\tau_{2(n-1)}, T]} W(C_{2(n-1)}, \tau_{2(n-1)+1}, \tau^*_{2n+1}(C_{2(n-1)+1}))$$

where the agency is not communicating in $[\tau_{2(n-1)}, \tau_{2(n-1)+1},)$, not communicating in $[\tau_{2(n-1)+1}, \tau_{2n+1}(C_{2(n-1)+1}))$ by previous step, and communicating in $\tau^*_{2n+1}(C_{2(n-1)+1}), T]$. Clearly the choice of $\tau_{2(n-1)+1}$ is irrelevant, we can just set it equal to its lower bound, and the problem can be rewritten as:

$$\max_{\tau^*_{2n+1}(C_{2(n-1)})\} \in [\tau_{2(n-1)}, T]} W(C_{2(n-1)}, \tau^*_{2n+1}(C_{2(n-1)}))$$

This problem is now isomorphic to the one solved in step 1, and by lemma(3) we know that $\frac{p^2}{(p+P(\tau_{2(n-1)}))^3} p_\varepsilon > \frac{c}{T-\tau_{2(n-1)}}$. Since the solution did not depend on any specific value of n, as long as $n > 1$ we can repeat the same steps n times until the problem becomes:

$$\max_{\tau^*_{2n+1} \in [0,T]} W(\tau^*_{2n+1})$$

where all choices of cut-points have been optimally set to the lower bound, that is $\tau^*_i = 0$ for $i < 2n+1$. Then the problem is equal to the problem in proposition (1). By lemma (3) we know that $\frac{p^2}{(p+P_\theta)^3} p_\varepsilon > \frac{c}{T}$, therefore by proposition (1) the optimal solution will be to set $\tau^*_{2n+1} = \tau^* > 0$. $\square$

Now the proof of the statement:

Step 1

Fix some $n \geq 1$, and consider solving the following problem:

$$\max_{\tau_{2n+1} \in [\tau_{2n}, T]} W(C_{2n}, \tau_{2n+1})$$

if $\frac{p^2}{(p+P(\tau_{2n}))^3} p_\varepsilon > \frac{c}{T-\tau_{2n}}$, then by lemma (4) the statement is proved. Therefore suppose that $\frac{p^2}{(p+P(\tau_{2n}))^3} p_\varepsilon \leq \frac{c}{T-\tau_{2n}}$. By proposition (1) it will be optimal to set τ^*_{2n+1} to its lower bound, that is $\tau^*_{2n+1} = \tau_{2n}$, so that the agency is always communicating in $[\tau_{2n}, T]$.

Step 2

Given the optimal choice in step 1, we can consider one more interval in our problem, that is solving:

$$\max_{\tau_{2n+1} \in [\tau_{2(n-1)+1}, T]} W(C_{2(n-1)+1}, \tau_{2n+1})$$

where $\tau_{2n+1} = \tau_{2n}$. The agency is communicating in $[\tau_{2(n-1)+1}, \tau_{2n} = \tau_{2n+1})$ and it is also communicating in $[\tau_{2n} = \tau_{2n+1}, T]$. Hence the choice of $\tau_{2n} = \tau_{2n+1}$ is irrelevant. By convention we will set $\tau_{2(n-1)+1} = \tau_{2n} = \tau_{2n+1}$.

Step 3

If $n > 1$, consider now solving the problem:

$$\max_{\tau_{2n+1} \in [\tau_{2(n-1)}, T]} W(C_{2(n-1)}, \tau_{2n+1})$$

where $\tau_{2n+1} = \tau_{2n} = \tau_{2(n-1)+1}$. The problem is isomorphic to the problem solved in step 1. Since the solution in step 1 did not depend on the choice of n, if $\frac{p^2}{(p+P(\tau_{2(n-1)}))^3} p_\varepsilon > \frac{c}{T-\tau_{2(n-1)}}$, then by lemma (4) the statement is proved, otherwise I can repeat the three steps n times until the problem becomes:

$$\max_{\tau_{2n+1} \in [0,T]} W(\tau_{2n+1})$$

where all τ_i's, $i < 2n+1$ have been optimally set to the lower bound 0. Then the problem is equal to the problem in proposition (1), and the optimal solution will be to set $\tau^*_{2n+1} = \tau^* > 0$ if $\alpha > \frac{c}{T}$, otherwise $\tau^*_{2n+1} = \tau^* = 0$. $\square$

A.5 Proof of proposition 5, Comparative Statics

Proof. From theorem 1, we know that when $\alpha > \frac{c}{T}$, where $c = \frac{7}{3}$, $\tau^* = -\frac{2}{\alpha} + \sqrt{\frac{5}{3\alpha^2} + \frac{T}{\alpha}}$. Clearly τ^* is increasing in T. The effect of the other parameters on τ^* can be understood by studying the sign of $\frac{\partial \tau^*}{\partial \alpha}\frac{\partial \alpha}{\partial \circ}$.

$$\frac{\partial \tau^*}{\partial \alpha} = \frac{1}{2\alpha^2}\left[4 - \frac{10/3 + \alpha T}{(5/3 + \alpha T)^{1/2}}\right] = \begin{cases} > 0 & \text{for } \alpha \in \left(\frac{c}{T}, \gamma\frac{c}{T}\right) \\ < 0 & \text{for } \alpha > \gamma\frac{c}{T} \end{cases}$$

where $\gamma \equiv \left(2 + 4\sqrt{\frac{3}{7}}\right) > 1$. Remembering that $\alpha = \frac{p^2}{(p+P_\theta)^3}p_\varepsilon$:

$$\frac{\partial \alpha}{\partial P_\theta} < 0, \quad \frac{\partial \alpha}{\partial p_\varepsilon} > 0, \quad \frac{\partial \alpha}{\partial p} = \frac{p\, p_\varepsilon(2P_\theta - p)}{(p + P_\theta)^4} = \begin{cases} > 0 & \text{for } p < 2P_\theta \\ < 0 & \text{for } p > 2P_\theta \end{cases}$$

Comparative statics for p_ε

α is monotonically increasing function of p_ε and it has value 0 for $p_\varepsilon = 0$. Therefore for all p, P_θ, T, there exists p'_ε such that $\alpha > \frac{c}{T}$ for $p_\varepsilon > p'_\varepsilon$; and there exists $p''_\varepsilon > p'_\varepsilon$ such that $\alpha > \gamma\frac{c}{T}$ for $p_\varepsilon > p''_\varepsilon$. Therefore:

- for $p_\varepsilon \in (p'_\varepsilon, p''_\varepsilon]$, $\frac{\partial \tau^*}{\partial \alpha} > 0$ (with equality at the right extremum), $\frac{\partial \alpha}{\partial p_\varepsilon} > 0$, therefore $\frac{\partial \tau^*}{\partial p_\varepsilon} > 0$ (with equality at the right extremum);
- for $p_\varepsilon > p''_\varepsilon$, $\frac{\partial \tau^*}{\partial \alpha} < 0$, $\frac{\partial \alpha}{\partial p_\varepsilon} > 0$, therefore $\frac{\partial \tau^*}{\partial p_\varepsilon} < 0$.

Comparative statics for P_θ

α is a monotonically decreasing function of P_θ and $\alpha \to 0$ as $P_\theta \to \infty$, therefore a necessary condition for $\alpha > \frac{c}{T}$ is that $\alpha(P_\theta = 0) = \frac{p_\varepsilon}{p} > \frac{c}{T}$. Then there exists $\tilde{P}_\theta$ such that, for $P_\theta < \tilde{P}_\theta$, $\alpha > \frac{c}{T}$. Consider the parameter values defined above, then two cases can happen:

i. $\alpha(P_\theta = 0) = \frac{p_\varepsilon}{p} > \gamma\frac{c}{T}$

   Then there exists P'_θ such that $\alpha > \gamma\frac{c}{T}$ for $P_\theta \in [0, P'_\theta]$ (with equality at the right extremum), and $\alpha < \gamma\frac{c}{T}$ for $P_\theta \in (P'_\theta, \tilde{P}_\theta)$. Therefore:

   - for $P_\theta \in [0, P'_\theta]$, $\frac{\partial \tau^*}{\partial \alpha} < 0$ (with equality at the right extremum), $\frac{\partial \alpha}{\partial P_\theta} < 0$, therefore $\frac{\partial \tau^*}{\partial P_\theta} > 0$ (with equality at the right extremum);
   - for $P_\theta \in (P'_\theta, \tilde{P}_\theta)$, $\frac{\partial \tau^*}{\partial \alpha} > 0$, $\frac{\partial \alpha}{\partial P_\theta} < 0$, therefore $\frac{\partial \tau^*}{\partial P_\theta} < 0$.

ii. $\alpha(P_\theta = 0) = \frac{p_\varepsilon}{p} \leq \gamma\frac{c}{T}$
Then for $P_\theta \in [0, \tilde{P}_\theta)$, $\frac{\partial\tau^*}{\partial\alpha} > 0$, $\frac{\partial\alpha}{\partial P_\theta} < 0$, therefore $\frac{\partial\tau^*}{\partial P_\theta} < 0$

Comparative statics for p

α is a hump-shaped function of p, tends to 0 both as $p \to 0$ and as $p \to \infty$, and has a maximum in $p^* \equiv 2P_\theta$. A necessary condition for $\alpha > \frac{c}{T}$ is that $\alpha(p^*) > \frac{c}{T} \implies \frac{p_\varepsilon}{P_\theta} > \frac{63}{4T}$. Let $P_\theta, p_\varepsilon, T$ satisfy the previous inequality, then there exist $\underline{p}, \bar{p}$, such that, for $p \in (\underline{p}, \bar{p})$, $\alpha > \frac{c}{T}$.

Let the parameters be such that, as defined before, $\alpha > \frac{c}{T}$. Two cases can happen:

i. $\alpha(p^*) > \gamma\frac{c}{T}$
Then there exists p' and p'' such that $\alpha > \gamma\frac{c}{T}$ for $p \in (p', p'')$, with equality at the extrema. We will have:

- for $p \in (\underline{p}, p']$, $\frac{\partial\tau^*}{\partial\alpha} > 0$ (with equality at the right extremum), $\frac{\partial\alpha}{\partial p} > 0$, therefore $\frac{\partial\tau^*}{\partial p} > 0$ (with equality at the right extremum);
- for $p \in (p', 2P_\theta]$, $\frac{\partial\tau^*}{\partial\alpha} < 0$, $\frac{\partial\alpha}{\partial p} > 0$ (with equality at the right extremum), therefore $\frac{\partial\tau^*}{\partial p} < 0$ (with equality at the right extremum);
- for $p \in (2P_\theta, p'']$, $\frac{\partial\tau^*}{\partial\alpha} < 0$, $\frac{\partial\alpha}{\partial p} < 0$ (with equality at the right extremum), therefore $\frac{\partial\tau^*}{\partial p} > 0$ (with equality at the right extremum);
- for $p \in (p'', \bar{p})$, $\frac{\partial\tau^*}{\partial\alpha} > 0$, $\frac{\partial\alpha}{\partial p} < 0$, therefore $\frac{\partial\tau^*}{\partial p} < 0$.

ii. $\alpha(p^*) \leq \gamma\frac{c}{T}$

- for $p \in (\underline{p}, 2P_\theta]$, $\frac{\partial\tau^*}{\partial\alpha} > 0$, $\frac{\partial\alpha}{\partial p} > 0$ (with equality at the right extremum), therefore $\frac{\partial\tau^*}{\partial p} > 0$;
- for $p \in (2P_\theta, \bar{p})$, $\frac{\partial\tau^*}{\partial\alpha} > 0$, $\frac{\partial\alpha}{\partial p} < 0$, therefore $\frac{\partial\tau^*}{\partial p} < 0$;

□

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