# Informational Effects of Monetary Policy

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**Abstract**

We analyze a simplified New-Keynesian model with an unobserved aggregate cost-push shock where firms and the central bank have different information about the shock. We consider a linear policy rule where a pure inflation targeting central bank decides how much to react to the shock given its information. In this framework we show that monetary policy serves both an *allocational* and an *informational* role, the latter coming from firms extracting information on the aggregate shock from the monetary policy tool. When the informational role is present, optimal monetary policy is more cautious, that is, it responds less to the shock than the perfect information benchmark. A more cautious reaction to the shock implies that firms use more effectively their private information and the endogenous information coming from the aggregate price in order to infer about the shock.

## 1 Introduction

Standard models used for monetary policy analysis usually assume that private households, firms and the policy maker all share a common information set, by perfectly observing the realizations of the relevant shocks. Yet, in practice, policy decisions are taken in an environment plagued with uncertainty about those shocks, and where agents hold different views about them.

Allowing for incomplete and heterogeneous information can significantly affect the monetary policy transmission mechanism. In models with complete information, monetary policy decisions only have an *allocational* effect: monetary policy decisions in response to demand or supply shocks directly affect economic outcomes. If instead economic agents observe only imperfect and heterogeneous signals of the shocks' realizations (and not their true values), monetary policy also becomes a channel through which the central bank affects the way private agents learn about the unknown state of the world: monetary policy decisions have an *informational* role.

The objective of this paper is to analyze how and why the optimal monetary policy changes in presence of imperfect information. We consider a simplified version of the

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New Keynesian model similar to the one considered in Walsh (2007, 2008). Firms set prices à la Calvo and are subject to an aggregate cost-push shock; firms and the central bank do not observe current aggregate shocks until the period is over, but they receive an idiosyncratic signal on the realization of the shock. The demand side of the model is highly stylized, specifically we assume that the output gap depends on two components: the first one is set directly by the monetary authority (through the policy instrument), the second one is a shock that the central bank cannot control; in setting the policy instrument the policy maker uses its own information about the aggregate cost-push shock, with the objective of minimizing fluctuations in inflation. Thus the output-gap reflects imperfectly the information of the central bank.

As common in the literature stemming from Hayek (1945) analysis the aggregate price, observed after pricing decisions of the firms, reflects dispersed information in the economy about the aggregate shock and constitutes a valuable source of learning about unknown states of the world. In particular, since it depends both on firms' behavior and on monetary policy decisions, price aggregates both the diffused information of the agents and the information of the central bank.

We show that the informational role of monetary policy is twofold. On the one hand, (i) since changes of the policy instruments depend on the information that the central bank has about the shocks, they provide implicitly a signal on the information of the central bank even in the absence of an explicit statement, hence affecting the beliefs of market participants. On the other hand, (ii) when the signalling channel of monetary policy decisions interact with the public signal deriving from prices in the economy and the private signal that each firm has on the relevant shock, a fundamental issue can arise: agents may rely less on their own private information and on the public signal on prices, with the effect that prices will reflect more the private information of the central bank.

Hence, in our model the central bank faces two types of trade-off. On the one hand, it has to balance the allocational channel – direct effect on variability of output gap and inflation – and the signaling one – indirect effect on firms beliefs about the aggregate shock. On the other hand, it has to consider the trade-off between information obtained from policy decisions (which reflect knowledge of the central bank) and information obtained from prices (which reflect dispersed information in the economy).

We assume that the central bank follows a policy rule to set the output gap, and we study how to choose the stance of this policy rule (i.e., how aggressively the central bank reacts to changes in expectations of the cost-push shock) in order to minimize expected inflation volatility. This stance is common knowledge in the economy. Private agents are Bayesian, hence they use all the available information (from their own private signal, market prices and central bank policy) when they form their beliefs.

The first result is that optimal policy depends on who has informational frictions between the central bank and the private agents: when the latter are perfectly informed while the central bank is not, optimal policy is equal to the perfect information benchmark (certainty equivalence holds); when agents are imperfectly informed the optimal monetary policy is different with respect to the perfect information benchmark (certainty equivalence does not hold). In this case policy parameters affect expectations, and the central bank has to take into account how information embedded in monetary policy is handled by the agents. In particular, when both are imperfectly informed, the relative amount of knowledge about the shocks matters for the optimal policy, as it determines the relevance of the different learning channels and how these translate into inflation volatility.

The second result (and main contribution of our work) is that aggressive monetary policies (high, in absolute value, coefficients of policy rule) have large allocational and informational effects. In this case firms rely more on information obtained from policy actions and less on the public signal obtained from prices (in the words of Morris and Shin (2005), there is less space for dissent in the economy). Conversely, lenient policy decisions imply almost no information to be learnt from monetary policy but a lot of information to be learnt from prices.

The structure of the work is as follows. In the next subsection we explain in which ways our work is related to different branches of the literature and how it is different from previous work. Section 2 introduces the model while section 3 defines and solves for optimal monetary policy in the perfect information economy benchmark. In section 4 we analyze and comment two particular features of the imperfect information economy, the role of price as aggregator of information and the channel through which monetary policy takes its informational role. Section 5 derives the equilibrium beliefs and inflation for the imperfect information economy and section 6 studies optimal monetary policy. Section 7 concludes.

### 1.1 Related Literature

We refer to different strands of the literature that explore informational asymmetries in macro models. The first strand specifically takes into account the transmission of information by a central bank either directly, through announcement, or indirectly, through monetary policy. A first example of this line of research is Canzoneri, Henderson and Rogoff (1983), in which the authors show that monetary policy can be designed so to convey information to agents if these have a type of bounded rationality which prevents learning from other sources. Our work is most closely related to that of Walsh (2007) and Walsh (2008) (that provide also the starting point for our modelization). In Walsh (2008) a central bank has to choose the optimal monetary policy under imperfect information about a cost-push shock and a demand shock and the analysis focuses on whether it is welfare beneficial to directly disclose central bank information or not. Walsh (2007) poses a similar question than Walsh (2008) with the added feature that instead of comparing two opposite communication regimes (the central bank is transparent about its information or not), the central bank chooses the optimal degree of dissemination of its information across economic agents. We share with these works the informative role of monetary policy, however our main question is not on communication policy per se, but on how informative frictions shape the optimal monetary policy. This allows us to derive more clear implication for monetary policy in imperfect information settings that are not explored in these works. In addition, even though our framework will feature only one shock, we take into account the role of prices as aggregating dispersed information and how these influence monetary policy, a channel that is not present in previous works.

The informative role of monetary policy is also studied in Baeriswyl and Cornand (2010b) and Baeriswyl and Cornand (2010a). While their model has no direct effect of monetary policy on learning, we allow for monetary decisions to affect beliefs, both directly through monetary policy and indirectly through learning from prices.

Nimark (2008) studies monetary policy in a new-keynesian model with asymmetric information, however in his model there is no informative role of monetary policy since by assumption it only conveys redundant information to agents. Melosi (2012) builds on Nimark (2008) but allows for an informative role of monetary policy. He calibrates the model on U.S. data to show that the signaling effect of monetary policy is empirically relevant. However it does not focus on optimal monetary policy.

Differently than previous papers Hoerova, Monnet and Temzelides (2012) asks the question of when communication by a central bank can be credible. In their model credible communication of the central bank can be obtained only when monetary policy transmits information and not when the central bank simply announces its information, since the former is costly for the central bank to manipulate.

We are also related to the large and growing strand of literature that studies informational frictions in monetary models. Brainard (1967) is the first study that introduces imperfect information deriving a result of policy cautiousness. Its result is though based on uncertainty about structural parameters, and not about shocks, as we have in our work. More recently Hellwig (2005) studies the effect of public information in New Keynesian economies, obtaining that more public information is always welfare improving in this setup. Amato and Shin (2005), Lorenzoni (2010) and Angeletos and La'o (2011) study optimal monetary policy with informational frictions but they do not have an informational role of monetary policy. Aoki (2003) studies optimal monetary policy in a new-keynesian model where agents are perfectly informed while the central bank only obtains noisy information about the shocks through the observation of endogenous realized variables. The work shows how estimation and control problem cannot be separated, a point analyzed in a more general setup in Svensson and Woodford (2004).

Regarding the informative role of prices, there is a long literature that goes back to Hayek (1945), or more recently with Grossman and Stiglitz (1980) and Hellwig and Venkateswaran (2011). We share with this literature the feature of prices aggregating dispersed information in the economy.

We are also finally related to the branch of the literature concerning the role of public vs. private information in agents' beliefs, analyzed in Vives (1993) and Miccoli (2012) in a dynamic setup, while Morris and Shin (2002), Angeletos and Pavan (2007) and Amador and Weill (2010) study welfare implication of public information in static economies with informational frictions.

# 2 Model

Following Walsh (2007) we consider a simplified new-keynesian model with firms and a central bank. The two following subsections define the problem of the firm and the monetary policy rule. The others describe the information structure of the economy and the problem of the central bank.

## 2.1 Firms

The economy is populated by a continuum of firms $j$ in the unit interval $[0,1]$; each firm in period $t$ produces a differentiated product using an identical technology subject to an unobserved aggregate cost-push shock $\theta_t$. We assume that this shock is independently and identically distributed over time according to the following distribution

$$\theta_t \sim N(0, P_\theta^{-1}). \tag{1}$$

Firms face a Calvo-type fixed probability of adjusting their price in each period: every period a fraction $(1-\omega)$ of firms can change its price. If the firms $j$ in period $t$ has the possibility to change its prices, it will set them according to the following condition:

$$p^*_{j,t} = (1-\omega\beta)\sum_{n=0}^{\infty}(\omega\beta)^n\left(\mathbb{E}^j_t p_{t+n} + \varphi_{t+n} + \mathbb{E}^j_t\theta_{t+n}\right)$$

where $\beta$ is the discount factor, $\varphi_t$ is the real marginal cost, $p_{t+n}$ is the aggregate price at time $t+n$ and the operator $\mathbb{E}^j_t$ defines the expectations of firm $j$ in period $t$.

This expression can be rewritten as

$$p^*_{j,t} = (1-\omega\beta)\left(\mathbb{E}^j_t p_t + \varphi_t + \mathbb{E}^j_t\theta_t\right) + \omega\beta\mathbb{E}^j_t p^*_{j,t+1}, \tag{2}$$

where the real marginal cost, as customary in the New Keynesian model, is related to the output gap measure $x_t$ by

$$\varphi_t = \kappa x_t, \tag{3}$$

and $\kappa$ is the output elasticity of marginal costs.

Since only a fraction $(1-\omega)$ of firms can change its price, the aggregate price is given by

$$p_t = (1-\omega)\overline{p}_t^* + \omega p_{t-1} \tag{4}$$

where

$$\overline{p}_t^* = \int_0^1 p_{j,t}^* dj. \tag{5}$$

Note that in equation (2) we have $\mathbb{E}_t^j \theta_t$ and $E_t^j p_t$ because, differently from the standard New Keynesian model, we assume that the aggregate prices and the cost-push shock at time $t$ are not observed by firm $j$. Regarding in particular the term $\mathbb{E}_t^j \theta_t$, it should be noticed that even though firms have the same technology and are affected by the same aggregate shock, they differ in their knowledge about its realization.

## 2.2 Central bank

For simplicity and analytical tractability we assume the central bank controls directly the output gap, $x_t$, according to a policy rule that reacts directly to the expected realization of the cost push shock at time $t$, $\theta_t$.[1] We also assume that the output gap depends on an additional component that the central bank cannot control, $\varepsilon_t$. We call this component the monetary policy shock and we assume that it is independently and identically distributed over time according to the following distribution

$$\varepsilon_t \sim N(0, P_\varepsilon^{-1})$$

The output gap at time $t$, will be:

$$x_t = \delta \mathbb{E}_t^{CB} \theta_t + \varepsilon_t \tag{6}$$

where $\delta$ is the policy parameter chosen by the central bank and $\mathbb{E}_t^{CB} \theta_t$ defines the time-$t$ expectations of the central bank over the time-$t$ realization of the aggregate cost-push shock.

## 2.3 Firms' Information

Firms have imperfect information about the current aggregate price and the realization of the cost-push shock. In order to estimate these two variables they have to use all the information available to them.

[1] Usually monetary policy is formulated in terms of control over the nominal interest rate. However in the standard aggregate demand specification of the new keynesian model, there is a one-to-one mapping between the control of the nominal interest rate and the resulting output gap (for given expectations). Therefore, our approach that lets the central bank directly choose the output gap is a reduced form of the more general new keynesian framework.

In particular, concerning the expectation about the time-$t$ realization of the cost-push shock, each firm-$j$ has four sources of information: (i) the prior on the realization of $\theta_t$, given, for simplicity, by the true distribution of the cost-push shock (1); (ii) a private signal $s_t^j$ defined as

$$s_{j,t} = \theta_t + \phi_{j,t} \tag{7}$$

where $\phi_{j,t} \sim N(0, P_\phi^{-1})$ i.i.d. across firms and time. As common in the information literature $P_\phi$, the inverse of the variance, is the precision of the information, and amount of information is defined in units of precision.[2]As we use the convention that $\int \phi_{j,t} dj = 0$, aggregating the signal over all firms implies that $\int s_{j,t} dj = \theta_t$. Note that the realization of the signal is private to firm $j$, hence firm $i \neq j$ will only know $s_{i,t}$ but not $s_{j,t}$.[3]

Each firm $j$ also has two aggregate (common to all firms) sources of information: (iii) a noisy signal about the aggregate price

$$p_t' = p_t + \eta_t \tag{8}$$

where $\eta_t \sim N(0, P_\eta^{-1})$, i.i.d. across time and (iv) the realization of the output gap $x_t$. Note that the signal on the aggregate price reveals information about the cost push shock since it aggregates the disperse information of the firms about this shock. This signal can be interpreted as preliminary releases of period $t$ aggregate prices. Note that if $P_\eta \to 0$, then firms would observe perfectly the price. As it will be clear below, this would amount to perfectly observing the cost-push shock $\theta_t$. But as long as $P_\eta$ is bounded away from zero, perfect discovery of the shock never obtains. The realization of the output gap, instead, represents a noisy signal of the cost-push shock as it depends on the information of the central bank about this shock and the monetary policy shock $\varepsilon_t$. This particular signal is fundamental to our work and will be carefully analyzed in section 4.

Firms form expectations about the unobservable variables at time-t by aggregating all the four sources of information. In section 5 we will show how firms use these sources of information in order to obtain an estimate of the shock.

### 2.4 Central bank's Information

Differently from the firms we start by assuming that the central bank does not observe the signal on aggregate prices.[4] Therefore there are two sources of information about the aggregate cost-push shock: (i) the prior on the aggregate shock $\theta_t$, that for simplicity is assumed to be the same of the firms, and (ii) a private signal (observable only by the

[2]Throughout the paper we will use interchangeably higher precision, more precise information and higher knowledge.

[3]This private signal can be interpreted as the firm knowing something about the aggregate shock that is specific to the variety of the good that she produces.

[4]The possibility for the central bank of using the public signal on prices is left for future research.

central bank) on the cost-push shock:

$$\theta_t^{CB} = \theta_t + \mu_t \tag{9}$$

where $\mu_t \sim N(0, P_\mu^{-1})$. This signal can be interpreted as the process of collecting preliminary data about the economy and estimating the size of the realization of the cost-push shock by the central bank.

The central bank is Bayesian in its use of information. Normality of signals implies that the mean posterior belief of the central bank about the shock is:

$$\mathbb{E}_t^{CB}\theta_t = \frac{P_\mu}{P_\mu + P_\theta}\theta_t^{CB}.$$

## 2.5 Central bank's objective function

The central bank chooses $\delta$ in the policy rule (6) with the objective of minimizing ex-ante variance of inflation, that is:

$$\min_\delta \mathbb{E}_{-1} \sum_{t=0}^{\infty} \beta^t \pi_t^2 \tag{10}$$

where $\pi_t = p_t - p_{t-1}$. Note that this welfare criterion implies that we are solving a problem under commitment, that is, we abstract away from situations in which the central bank re-optimizes its choice given a particular realization of its information.

The ex-ante welfare criterion, together with the quadratic objective and normality assumptions, also implies that $\delta$ does not effectively communicate any information about the realization of the shocks, since ex-ante information depends only on the structural parameters of the model (precision of information) that are commonly known among firms and the central bank.

## 2.6 Timing

The timing of the model is the following:

1. At the beginning of the history, in period $-1$, the central bank announces $\delta$.

2. At the beginning of every period $t \geq 0$, a shock $\theta_t$ is realized; the central bank and each firm-j observe a private signal on the realization of the cost-push shock, respectively $\theta_t^{CB}$ and $s_{j,t}$; moreover, each firm-j observes the realization of the output gap, $x_t$, receives an endogenous signal about aggregate prices $p_t'$; a fraction $(1 - \omega)$ of firms sets the prices.

3. All actors in the economy learn the realization of $\theta_t$ and the aggregate price $p_t$.

At any point in time, the monetary policy rule, structural precision parameters of information and the $\delta$ announced by the central bank, are common knowledge among the actors in the economy.

## 3 Perfect Information Benchmark

Before analyzing the effects of dispersed information we solve the model under the perfect information assumption. Specifically, we assume that $\theta_t$ and $p_t$ are common knowledge at time $t$, but firms and the central bank yet do not know the realizations of future cost-push shocks and aggregate prices.

Given perfect information, the pricing equation for firm $j$ at any period $t$ is:

$$p^*_{j,t} = (1-\omega\beta)(p_t + \kappa x_t + \theta_t) + \omega\beta \mathbb{E}^j_t p^*_{j,t+1} \tag{11}$$

To solve the model we use a guess and verify approach. In particular, we guess that

$$p^*_{j,t+i} = Ap_{t+i-1} + B\theta_{t+i} + C\varepsilon_{t+i} \qquad \forall i \geq 0 \tag{12}$$

Note that we do not need to assume that $\varepsilon_t$ is directly observable, as firms know $x_t$, $\delta$ and $\theta_t$; given these variables they can obtain $\varepsilon_t$ directly from equation (6). By using the guess for $i = 1$ we have that:

$$\mathbb{E}^j_t p^*_{j,t+1} = Ap_t + B\mathbb{E}^j_t \theta_{t+1} + C\mathbb{E}^j_t \varepsilon_{t+1}.$$

Since at the beginning of time-t firms' priors on $\theta_{t+1}$ and $\varepsilon_t$ are $\theta_{t+1} \sim N(0, P_\theta^{-1})$ and $\varepsilon_t \sim N(0, P_\varepsilon^{-1})$, we have that

$$\mathbb{E}^j_t p^*_{j,t+1} = Ap_t. \tag{13}$$

From the guess at $i = 0$ instead we have that:

$$\int p^*_{j,t} dj = Ap_{t-1} + B\theta_t + C\varepsilon_t$$

that can be used to derive the aggregate price in period $t$:

$$p_t = ((1-\omega)A + \omega)p_{t-1} + (1-\omega)B\theta_t + (1-\omega)C\varepsilon_t. \tag{14}$$

By substituting (13) and (14) into (11), and since under perfect information $x_t = \delta\theta_t + \varepsilon_t$, we obtain

$$p^*_{j,t} = (1-\omega\beta+\omega\beta A)\left[((1-\omega)A + \omega)p_{t-1} + (1-\omega)B\theta_t + (1-\omega)C\varepsilon_t\right] + (1-\omega\beta)\kappa(\delta\theta_t + \varepsilon_t) + (1-\omega\beta)\theta_t.$$

Mapping the last equation into our guess gives the following fixed points:

$$A = (1 - \omega\beta + \omega\beta A)((1 - \omega)A + \omega) \tag{15}$$

$$B = (1 - \omega\beta + \omega\beta A)(1 - \omega)B + (1 - \omega\beta)(\kappa\delta + 1) \tag{16}$$

$$C = (1 - \omega\beta + \omega\beta A)(1 - \omega)C + (1 - \omega\beta)\kappa \tag{17}$$

Note that equation (15) can be solved separately since it does not depend on the other coefficients. We obtain the following quadratic equation in $A$:

$$A^2 - \left[1 + \frac{1 - \omega\beta}{\beta(1 - \omega)}\right] A + \frac{1 - \omega\beta}{\beta(1 - \omega)} = 0. \tag{18}$$

Even though both $A_1 = 1$ and $A_2 = \frac{1-\omega\beta}{\beta(1-\omega)}$ are solution of equation (18), only $A_1$ implies a stationary path for inflation. In fact substituting $A_2$ into equation (14) we obtain

$$p_t = \frac{1}{\beta} p_{t-1} + (1 - \omega)B\theta_t + (1 - \omega)C\varepsilon_t$$

and since $\frac{1}{\beta} > 1$, $p_t$ would follow an explosive path.

We therefore discard solution $A_2$ and we focus on $A_1 = 1$. Substituting this solution in (13) we obtain

$$p_t = p_{t-1} + (1 - \omega)B\theta_t + (1 - \omega)C\varepsilon_t$$

or, in terms of inflation

$$\pi_t = (1 - \omega)B\theta_t + (1 - \omega)C\varepsilon_t.$$

Using this result we can rewrite the fixed points for $B$ and $C$ as:

$$B = \frac{1 - \omega\beta}{\omega}(\kappa\delta + 1) \tag{19}$$

$$C = \frac{1 - \omega\beta}{\omega}\kappa \tag{20}$$

Our guess is hence verified and we have that, under perfect information,

$$p^*_{j,t+i} = p_{t+i-1} + \frac{(1 - \omega\beta)}{\omega}(\kappa\delta + 1)\theta_{t+i} + \frac{(1 - \omega\beta)}{\omega}\kappa\varepsilon_{t+i} \qquad \forall i \geq 0 \tag{21}$$

and aggregate inflation is given by:

$$\pi_t = \frac{(1 - \omega)(1 - \omega\beta)}{\omega}(\kappa\delta + 1)\theta_t + \frac{(1 - \omega)(1 - \omega\beta)}{\omega}\kappa\varepsilon_t \tag{22}$$

A central bank with a strict inflation targeting objective, i.e. it minimizes variance of inflation, can achieve its objective by completely offsetting the aggregate cost-push shock.We summarize the results of this section in the following proposition.

**Proposition 1** *With perfect information about the aggregate shock, the optimal monetary policy is $\delta^* = -1/\kappa$, that is, to completely offset the aggregate cost-push shock.*

**Proof.** Under strict inflation targeting, the central bank solves problem (10), subject to (22). From the FOC of this constrained minimization problem we obtain $\delta^* = -1/\kappa$. ■

# 4 The Imperfect Information economy

We consider now the case in which firms cannot observe at the beginning of period-$t$ neither the realization of the cost-push shock, nor the aggregate price. As first step we will investigate how firms extract information about the cost-push shock through the observation the noisy signal on aggregate price $p'_t$ and the observation of the output gap, $x_t$.

## 4.1 Endogenous signal on the cost-push shock from the aggregate price

The signal on aggregate price constitutes a valuable source of information for firms, since it aggregates all information in the economy. In a scenario where aggregate prices are perfectly observed, the non atomistic structure of firms would imply that $p_t$ exactly reveals the aggregate shock. However, if $\theta_t$ and aggregate prices are imperfectly observed, the signal on prices provides additional information about the cost-push shock.[5]

Since aggregate prices are determined endogenously from the model, we start by guessing that in equilibrium prices are a linear combination of the four observable variables, $s_{j,t}, x_t, p'_t, p_{t-1}$. However, since $p'_t$ also depends on the other three state variable, we define an additional variable $\tilde{p}_t$ as the component of the price signal $p'_t$ that is hortogonal to the other three variables. Given this new variable, we guess that the optimal price for firm $j$ at time $t$ is a linear time invariant function of the four state variables of the economy:

$$p^*_{j,t} = \alpha s_{j,t} + \rho x_t + \varphi \tilde{p}_t + \gamma p_{t-1} \tag{23}$$

where $\tilde{p}_t$ is the endogenous signal that we want to find, and $\alpha, \rho, \varphi, \gamma$ are yet to be determined coefficients. By the definition of aggregate prices (equation (4)), we obtain:

$$p_t = (1-\omega)\left(\alpha\theta_t + \rho x_t + \varphi\tilde{p}_t\right) + \left((1-\omega)\gamma + \omega\right) p_{t-1}.$$

Since the price signal is the additional information on $\theta_t$ coming from the aggregate price that is not already in the information set of the firms, we rearrange the signal $p'_t = p_t + \eta_t$ so that it depends only on not commonly observed state variables. This is

[5] In our setup the imperfect observation of aggregate prices is assumed, however it can also be microfounded following the reasoning in Lorenzoni (2009).

obtained by rewriting $p'_t$ as:

$$\frac{p'_t-(1-\omega)\rho x_t-(1-\omega)\varphi\tilde{p}_t-((1-\omega)\gamma+\omega)\,p_{t-1}}{(1-\omega)\alpha}=\theta_t+\frac{\eta_t}{(1-\omega)\alpha}.$$

Now the rearranged signal defined above is exactly the orthogonal information firms extract from the aggregate price and by imposing the fixed point:

$$\tilde{p}_t\equiv\frac{p'_t-(1-\omega)\rho x_t-(1-\omega)\varphi\tilde{p}_t-((1-\omega)\gamma+\omega)\,p_{t-1}}{(1-\omega)\alpha}$$

we obtain the endogenous signal from the aggregate price:

$$\tilde{p}_t=\theta_t+\frac{\eta_t}{(1-\omega)\alpha}. \tag{24}$$

Note that the endogenous information diffusion through prices is evident in (24) through the presence of $\alpha$. In fact, the precision of this signal (conditional on $\theta_t$) is $(1-\omega)^2\alpha^2P_\eta$. $\alpha$ represents the role of private information in the pricing decision and so, the more prices reflect the private information of the firms (higher $\alpha$), the more the aggregate price will transmit the disperse information and, therefore, the more precise will be the signal from prices about $\theta_t$ (higher $(1-\omega)^2\alpha^2P_\eta$). This learning channel is one of the main contribution of this work with respect to the previous literature.

## 4.2 Informational Role of Monetary Policy

When firms set their price, they can observe the output gap given by the monetary policy rule $x_t=\delta\mathbb{E}_t^{CB}\theta+\varepsilon_t$. Since the output gap depends on central bank's information about the realization of the shock $\theta_t$, it constitutes a source of information for the firms. This is the informative role of monetary policy. Note that, if firms and the central bank shared the same information (namely, if they have the source of information), than monetary policy would not communicate new information to the firms. It is the heterogeneity of information sources between firms and the central bank that creates the informational role of monetary policy.

Given that the central bank only has its private information signal $\theta_t^{CB}$ and the common prior on $\theta_t$, its beliefs are given by $\mathbb{E}_t^{CB}\theta_t=\frac{P_\mu}{P_\theta+P_\mu}\theta_t^{CB}$. Since precision parameters are common knowledge, the firms rearrange the signal coming from the output gap in order to obtain an unbiased signal about $\theta_t$ defined as:

$$\tilde{x}_t\equiv\frac{x_t}{\delta}\frac{P_\theta+P_\mu}{P_\mu}=\theta_t^{CB}+\frac{\varepsilon_t}{\delta}\frac{P_\theta+P_\mu}{P_\mu}$$

whose precision is given by $P_x\equiv\left[\frac{1}{P_\mu}+\frac{1}{\delta^2P_\varepsilon}\left(\frac{P_\theta+P_\mu}{P_\mu}\right)^2\right]^{-1}$. When monetary policy is used as a source of information, then the policy parameter $\delta$ directly affects the information available to the firms. It is straightforward to see that $\frac{\partial P_x}{\partial|\delta|} > 0$, therefore an higher (in absolute value) $\delta$, the more precise the information coming from the monetary policy. The reason for this is clear: for higher values of $\delta$ the observed monetary policy depends more on the beliefs of the central bank than on the exogenous monetary policy shock $\varepsilon_t$. Therefore an higher (in absolute value) $\delta$ implies that monetary policy communicates more precisely the beliefs of the central bank and so its informativeness for the firms. Note that the sign of $\delta$ is irrelevant for the amount of information it communicates, since $P_x$ only depends on $\delta^2$. Intuitively, signal precision depends on the variance of the distribution, and this is sign invariant.

Differently than a perfect information model, in this model monetary policy not only it affects prices because it offsets the effect of the cost-push shock (the *allocational effect*), but also it also influences the information available to the firms, therefore their beliefs on the shock and how they will react to it. This new role of monetary policy, which we define the *informational effect* is the main object of analysis of this work. Even though the informational role of monetary policy is present in some previous paper in the literature (for instance Walsh (2007)), this is the first study that tries to exactly understand how the informational role influences monetary policy. In particular, how the two roles of monetary policy interact in determining the optimal policy is the main question that we want to answer.

# 5 Solving the model

Before turning to the main question of this work, optimal monetary policy under imperfect information, we need to specify period $t$ firms' beliefs and how they influence price setting and equilibrium inflation. The first part of this section deals with firms' learning process, the second part defines the price setting equilibrium given the monetary policy parameter $\delta$. The third part analyzes properties of the equilibrium, while the last part confronts how monetary policy influences equilibrium inflation in the imperfect information model with respect to the perfect information benchmark.

## 5.1 Beliefs updating

Firms use all available information when estimating $\theta_t$. This comes from their own private signal $s_{j,t}$, the information coming from aggregate price $\tilde{p}_t$ and the signal coming from the output gap $\tilde{x}_t$. Using standard formulas for Bayesian updating with normal signals, the mean posterior belief of $\theta_t$ in period $t$ for firm $j$ is:

$$\mathbb{E}_t^j \theta_t = \lambda_s s_{j,t} + \lambda_x \tilde{x}_t + \lambda_p \tilde{p}_t \tag{25}$$

where:

$$\lambda_s = \frac{P_\phi}{P_\theta + P_\phi + P_x + (1-\omega)^2\alpha^2 P_\eta}, \tag{26}$$

$$\lambda_x = \frac{P_x}{P_\theta + P_\phi + P_x + (1-\omega)^2\alpha^2 P_\eta}, \tag{27}$$

$$\lambda_p = \frac{(1-\omega)^2\alpha^2 P_\eta}{P_\theta + P_\phi + P_x + (1-\omega)^2\alpha^2 P_\eta}. \tag{28}$$

The policy parameters $\delta$ affects directly expectations of the firms. In particular, note that by changing $\delta$ the central bank not only affects how much information firms extract from the policy rule, but also what is the relative weight firms put on each different source of information in order to create the posterior beliefs. The exact effect of a change in $\delta$ on relative role of information sources depends on equilibrium parameter $\alpha$, therefore we postpone to later in the paper its analysis.

## 5.2 Equilibrium Prices

Pricing decisions of the firms depend on the signal they have on the aggregate cost-push shock. Since the price signal is endogenous to their choice, how information determines prices has to be determined through a fixed point analysis. In particular we are looking for a partially revealing rational expectations equilibrium in which the aggregate price does not fully reveal the aggregate cost-push shock $\theta_t$. Borrowing from the finance literature in rational expectations equilibria we specify a linear price guess, as already defined in (23). In order to solve for the undetermined coefficients we use our price guess to find the two unknown objects of the pricing decision's equation (2), $\mathbb{E}_t^j p_t$ and $\mathbb{E}_t^j p^*_{j,t+1}$. We then plug these objects back in the pricing equation to obtain the coefficients to match with our guess. We have shown before that the aggregate price is given by:

$$p_t = (1-\omega)\left(\alpha\theta_t + \rho x_t + \varphi\tilde{p}_t\right) + \left((1-\omega)\gamma + \omega\right) p_{t-1} \tag{29}$$

so that $\mathbb{E}_t^j p_t = (1-\omega)\left(\alpha\mathbb{E}_t^j\theta_t + \rho x_t + \varphi\tilde{p}_t\right) + \left((1-\omega)\gamma + \omega\right) p_{t-1}$. In order to find the expectation of future price settings, we use our guess for period $t+1$:

$$p^*_{j,t+1} = \alpha s_{j,t+1} + \rho\tilde{x}_{t+1} + \varphi\tilde{p}_{t+1} + \gamma p_t$$

Taking expectations with respect to the information set of firm $j$ in period $t$ we have that $\mathbb{E}_t^j p^*_{j,t+1} = \gamma\mathbb{E}_t^j p_t$. This is so since all future shocks are assumed to have zero ex-ante mean. We can now substitute these elements into the pricing equation to obtain:

$$\begin{aligned} p^*_{j,t} &= [1-\omega\beta(1-\gamma)]\,\mathbb{E}_t^j p_t + (1-\omega\beta)(\mathbb{E}_t^j\theta_t + \kappa x_t) \\ &= [1-\omega\beta(1-\gamma)]\left[(1-\omega)\left(\alpha\mathbb{E}_t^j\theta_t + \rho x_t + \varphi\tilde{p}_t\right) + \left((1-\omega)\gamma + \omega\right) p_{t-1}\right] + (1-\omega\beta)(\mathbb{E}_t^j\theta_t + \kappa x_t) \end{aligned}$$

where in the second line we substituted for $\mathbb{E}_t^j p_t$. Using the expectation of firms about $\theta_t$ and our definition of $\tilde{x}_t$, we can finally derive the equations solving for the undetermined coefficients in our guess:

$$\alpha = [1 - \omega\beta(1-\gamma)]\,[(1-\omega)\alpha + (1-\omega\beta)]\,\lambda_s \tag{30}$$

$$\rho = [1 - \omega\beta(1-\gamma)]\left[(1-\omega)\alpha\lambda_x \frac{P_\theta + P_\mu}{\delta P_\mu} + (1-\omega)\rho\right] + (1-\omega\beta)(\lambda_x \frac{P_\theta + P_\mu}{\delta P_\mu} + \kappa) \tag{31}$$

$$\varphi = [1 - \omega\beta(1-\gamma)]\,[(1-\omega)\alpha\lambda_p + (1-\omega)\varphi] + (1-\omega\beta)\lambda_p \tag{32}$$

$$\gamma = [1 - \omega\beta(1-\gamma)]\,[(1-\omega)\gamma + \omega] \tag{33}$$

Note that monetary policy influences the aggregate price through the coefficient $\rho$, that is, through the output gap: this the standard allocational effect of monetary policy. However since $\delta$ influences the beliefs through the updating weights $\lambda$s, it also affects every single coefficient of the price guess. The way monetary influences beliefs and then equilibrium coefficients is the informational effect of monetary policy.

### 5.3 Properties of the equilibrium

There is no theoretical reason that forces the equilibrium to be unique, however the two following propositions will indeed show that the equilibrium consistent with stationary inflation is unique.

**Proposition 2** *The only solution consistent with a stationary path of inflation is characterized by $\gamma = 1$.*

**Proof.** Note that the fixed point for $\gamma$ is independent of all other equations, hence we can solve for this directly. By equation (33) we have that:

$$\begin{aligned}\gamma &= [1 - \omega\beta(1-\gamma)]\,[\gamma + \omega(1-\gamma)] \\ 0 &= \omega(1-\beta)(1-\gamma) + \omega\beta(1-\omega)(1-\gamma)^2\end{aligned}$$

This equation has two solutions: $\gamma_1 = 1$ and $\gamma_2 = 1 + \frac{1-\beta}{\beta(1-\omega)}$. By substituting $\gamma_2$ into the equation for the aggregate price (29) we obtain:

$$p_t = (1-\omega)\alpha\theta_t + (1-\omega)\rho x_t + (1-\omega)\varphi\tilde{p}_t + \frac{p_{t-1}}{\beta}$$

which, since $1/\beta > 1$, it defines a non stationary path for inflation. On the other hand by substituting $\gamma_1$ we have:

$$\begin{aligned}p_t &= (1-\omega)\alpha\theta_t + (1-\omega)\rho x_t + (1-\omega)\varphi\tilde{p}_t + p_{t-1} \\ \Longrightarrow \pi_t &= (1-\omega)\alpha\theta_t + (1-\omega)\rho x_t + (1-\omega)\varphi\tilde{p}_t\end{aligned}$$

which implies stationary inflation. ■

Using the result of proposition 2 we can rewrite the fixed point equations as:

$$\alpha = [(1-\omega)\alpha + (1-\omega\beta)]\,\lambda_s \tag{34}$$

$$\rho = [(1-\omega)\alpha + (1-\omega\beta)]\,\lambda_x \frac{P_\theta + P_\mu}{\delta P_\mu} + (1-\omega)\rho + (1-\omega\beta)\kappa \tag{35}$$

$$\varphi = [(1-\omega)\alpha + (1-\omega\beta)]\,\lambda_p + (1-\omega)\varphi \tag{36}$$

The following proposition states that the equilibrium is unique.

**Proposition 3** *For all parameters values, there exists a unique partially revealing rational expectations equilibrium, i.e. there exists unique $\hat{\alpha}, \hat{\rho}, \hat{\varphi}$ that solve the system of equations (34)-(36). Moreover $\hat{\alpha} \in [0, \frac{(1-\omega\beta)P_\phi}{P_\theta + \omega P_\phi + P_x}]$.*

**Proof.** Note that equations (35), (36) can be rewritten only in terms of $\alpha$ and the updating weights $\lambda$:

$$\rho = \frac{[(1-\omega)\alpha + (1-\omega\beta)]}{\omega}\lambda_x \frac{P_\theta + P_\mu}{\delta P_\mu} + \frac{(1-\omega\beta)}{\omega}\kappa \tag{37}$$

$$\varphi = \frac{[(1-\omega)\alpha + (1-\omega\beta)]}{\omega}\lambda_p \tag{38}$$

Since the updating weights are functions only of $\alpha$, then for a given $\alpha$, $\rho$ and $\varphi$ are uniquely determined. In order to determine $\alpha$ we rewrite (34) using the definition of $\lambda_s$:

$$\alpha = [(1-\omega)\alpha + (1-\omega\beta)]\,\frac{P_\phi}{P_\theta + P_\phi + P_x + (1-\omega)^2\alpha^2 P_\eta}$$

$$\alpha^3 = -\frac{P_\theta + \omega P_\phi + P_x}{(1-\omega)^2 P_\eta}\alpha + \frac{(1-\omega\beta)P_\phi}{(1-\omega)^2 P_\eta} \tag{39}$$

Where $P_x = \left[\frac{1}{P_\mu} + \frac{1}{\delta^2 P_\varepsilon}\left(\frac{P_\theta + P_\mu}{P_\mu}\right)^2\right]^{-1}$. The left hand side of equation (39) is strictly increasing in $\alpha$ and takes negative (positive) values for $\alpha < 0 (> 0)$. The right hand side is strictly decreasing in $\alpha$ and takes positive values for $\alpha = 0$. Then the left hand side and the right hand side will intersect only once, in the region where $\alpha > 0$, therefore there exists a unique $\hat{\alpha} > 0$ that solves equation (39). Moreover we can fix a higher bound on $\alpha$, $\bar{\alpha}$ simply by setting the right hand side equal to 0. This gives:

$$\bar{\alpha} = \frac{(1-\omega\beta)P_\phi}{P_\theta + \omega P_\phi + P_x} \tag{40}$$

■

Note that closed form solutions for the unique equilibrium coefficients exist but are analytically intractable, due to the cubic nature of the fixed point equation. However we can easily characterize some properties of $\hat{\alpha}$ which are going to be important to understand how monetary policy influences beliefs (note that only $\hat{\alpha}$ enters into the posterior beliefs of the firms. The following proposition provides comparative statics of $\hat{\alpha}$ with respect to $\delta$.

**Proposition 4** *$\hat{\alpha}$ is strictly decreasing in the absolute value of $\delta$*

**Proof.** $\delta$ influences only $P_x$ in equation (39). In particular it is easy to show that $P_x$ is strictly increasing in the absolute value of $\delta$. When $P_x$ increases the right hand side of equation (39) rotates clockwise around the intersection of the $y$ axis. Since the left hand side is strictly increasing in $\alpha$, this implies that as $\delta$ increases, the equilibrium value of $\alpha$ is strictly lower. ■

As we explained before in section 4, higher $\delta$ (in absolute value) imply that the monetary policy rule is transmitting more information. The equilibrium effect is that pricing decisions reflect less the private information of the firms (remember from equation (23) that $\alpha$ maps private information into firms' price). This is so because monetary policy alters the relative role of the different sources of information for the firms, as the next proposition clarifies.

**Proposition 5** *$\lambda_s$, the weight firms put on their private information source, is decreasing in the absolute value of the monetary policy parameter $\delta$.*

**Proof.** Since $\delta$ influences $\hat{\alpha}$ only through $P_x$, and we already know that $\frac{\partial P_x}{\partial |\delta|} > 0$, we can just study how a change in $P_x$ modifies $\hat{\alpha}$. By equation (39) we define $f(\hat{\alpha}, P_x) = \alpha^3 + \frac{P_\theta + \omega P_\phi + P_x}{(1-\omega)^2 P_\eta}\alpha - \frac{(1-\omega\beta)P_\phi}{(1-\omega)^2 P_\eta}$, and using the implicit function theorem on $f(\hat{\alpha}, P_x) = 0$ we have that

$$\frac{\partial \hat{\alpha}}{\partial P_x} = -\frac{\partial f / \partial P_x}{\partial f / \partial \hat{\alpha}} = -\frac{\hat{\alpha}}{3\hat{\alpha}^2 (1-\omega)^2 P_\eta + P_\theta + \omega P_\phi + P_x} < 0 \tag{41}$$

therefore

$$\begin{aligned} \frac{\partial \lambda_s}{\partial P_x} &= -\frac{P_\phi}{(P_\theta + P_\phi + P_x + (1-\omega)^2 \hat{\alpha}^2 P_\eta)^2} \left(1 + 2\hat{\alpha}(1-\omega)^2 P_\eta \frac{\partial \hat{\alpha}}{\partial P_x}\right) \\ &= -\frac{P_\phi}{(P_\theta + P_\phi + P_x + (1-\omega)^2 \hat{\alpha}^2 P_\eta)^2} \left(1 - \frac{2\hat{\alpha}^2 (1-\omega)^2 P_\eta}{3\hat{\alpha}^2 (1-\omega)^2 P_\eta + P_\theta + \omega P_\phi + P_x}\right) < 0 \end{aligned}$$

which is negative since the term in the parentheses is always positive. ■

When the central bank increases (in absolute value) its policy parameter $\delta$, firms' beliefs will be based more on the monetary policy signal than on their private source of information or on the aggregate price.[6] Monetary policy decisions influence therefore the relative weight of each source of information and therefore how information maps into

[6] A corollary of proposition 5 is that $\lambda_p$, the weight on the price signal, is increasing in the absolute value of $\delta$ and $\lambda_x$ is increasing in the absolute value of $\delta$.

pricing decision. This is the channel through which operates the *informational* effect of monetary policy. Note that by changing the expectations, monetary policy affects all equilibrium coefficients of the equilibrium price, since all depend on beliefs. However while it is clear what is the effect on $\alpha$ and $\varphi$ (the latter being an increasing function of $\alpha$), the effect on $\rho$ cannot be clearly signed and we need to rely on numerical computations.

### 5.4 Equilibrium inflation compared to perfect information benchmark

Before analyzing optimal monetary policy, it is useful to compare equilibrium inflation in the imperfect information economy with the equilibrium inflation that would arise in the perfect information economy. Given that in equilibrium by proposition 2 $\gamma = 1$, we can use equation (29) to derive inflation in any period $t$:

$$\pi_t = (1-\omega)\left[\alpha(\delta)\theta_t + \rho(\delta)x_t(\theta_t, \delta, \varepsilon_t) + \varphi(\delta)\tilde{p}_t(\theta_t, \alpha(\delta))\right] \tag{42}$$

where $\pi_t \equiv p_t - p_{t-1}$ and the dependence of coefficients and signals on $\theta_t$ and $\delta$ has been highlighted to provide a better comparison with inflation in the perfect information benchmark, that is:

$$\pi_t^{PI} = (1-\omega)[B(\delta)\theta_t + C\varepsilon_t]. \tag{43}$$

Under perfect information monetary policy influences inflation only through the coefficient on the cost push shock (in (43) only the coefficient $B$ depends on $\delta$): this is how the allocational effect of monetary policy operates in the perfect information economy: by decreasing $B$ monetary policy decreases the impact of the cost-push on inflation. With imperfect information the allocational and informational effect are intertwined: monetary policy parameter $\delta$ influences inflation directly through the output gap $x_t$, but since it also modifies beliefs, it influences also all other equilibrium coefficients and the signal coming from the aggregate price. Since $\delta$ has different effects on the equilibrium coefficients, it is not direct to ascertain how monetary policy will trade-off between the informational effect and the allocational effect. The central bank will choose $\delta$ such that it minimizes the effect of the shock through all equilibrium coefficients. How the central bank balances the two effects is going to be analyzed in the next section.

## 6 Optimal monetary policy

The objective of the central bank is to minimize ex-ante volatility of inflation. Using equation (42) by substituting $x_t$ and $\tilde{p}_t$ we obtain:

$$\pi_t = (1-\omega)\left[(\alpha + \rho\delta\frac{P_\mu}{P_\theta + P_\mu}\mu_t + \varphi)\theta + \rho(\delta\frac{P_\mu}{P_\theta + P_\mu} + \varepsilon_t) + \varphi\frac{\eta_t}{(1-\omega)\alpha}\right]$$

For clarity it will be useful to define $\tilde{\rho} \equiv \rho\delta\frac{P_\mu}{P_\theta+P_\mu}$. Now by squaring $\pi_t$ and taking unconditional expectations we obtain:

$$\mathbb{E}\left[{\pi_t}^2\right] = (1-\omega)^2\left[(\alpha+\tilde{\rho}+\varphi)^2\frac{1}{P_\theta}+\tilde{\rho}^2\frac{1}{P_x}+\frac{\varphi^2}{(1-\omega)^2\alpha^2P_\eta}\right] \tag{44}$$

Note that the unconditional expectation of $\pi_t$ is not time dependent since it is function only of precision of signals. Therefore the loss function that determines the welfare objective is:

$$L(\delta) = \frac{(1-\omega)^2}{1-\beta}\left[(\alpha(\delta)+\tilde{\rho}(\delta)+\varphi(\delta))^2\frac{1}{P_\theta}+\tilde{\rho}(\delta)^2\frac{1}{P_x(\delta)}+\frac{\varphi(\delta)^2}{(1-\omega)^2\alpha(\delta)^2P_\eta}\right] \tag{45}$$

The problem of the Central is therefore to minimize $L(\delta)$ subject to equilibrium coefficients $\alpha(\delta)$, $\tilde{\rho}(\delta)$ and $\varphi(\delta)$. This minimization problem has some non-standard features, which require additional care to be sure that a solution exists. In fact, the objective function is not necessarily everywhere convex in the choice variable $\delta$, hence the concave programming instruments cannot be used. Moreover, the choice set is not compact, since $\delta$ can take any value between plus and minus infinity, so that Weierstrass theorem cannot be invoked.

To overcome these problems, in what follows we show that the objective function defined in equation (45) is coercive, namely that $\lim_{|\delta|\to\infty} L = \infty$. It is well known that coercivity of a function implies that all its lower level sets are compact[7], so that we can apply Weierstrass theorem to an auxiliary minimization problem where the choice set is restricted to any non-empty lower level set, and obtain existence of a global minimum of this auxiliary problem; obviously, any solution to the auxiliary problem is also a solution to the original problem.

**Proposition 6** *The function $L(\delta)$ defined in equation (45) is coercive, namely $\lim\limits_{|\delta|\to\infty} L(\delta) = \infty$.*

**Proof.** Observe that the function $L(\delta)$ in equation (45) is the sum of three nonnegative terms; hence a sufficient condition for coercivity is that one of these three terms is coercive. We concentrate on $\tilde{\rho}^2\frac{1}{P_x}$. First of all, note that $\lim_{|\delta|\to\infty} P_x = P_\mu$, which is typically a finite value; moreover, $\tilde{\rho}^2$ is proportional to $(\rho\delta)^2$. From equation (37), we get:

$$(\rho\delta)^2 = \left(\frac{[(1-\omega)\alpha+(1-\omega\beta)]}{\omega}\lambda_x\frac{P_\theta+P_\mu}{P_\mu}+\frac{(1-\omega\beta)}{\omega}\kappa\delta\right)^2 \tag{46}$$

It is easy to see that the first addend goes to some finite term as $|\delta|\to\infty$: as shown in Proposition 3, $\alpha$ is bounded between zero and an upper bound that depends on $\delta$ only because it is a continuous function of $P_x$ (see equation (40)); moreover, also $\lambda_x$ depends on

[7] A lower level set of the function $L(\delta)$ is given by $\{\delta : L(\delta) \leq r\}$, for some real number $r$.

$\delta$ (continuously) only through $P_x$ (see equation (27)). Hence the second addend dominates, which shows that $(\rho\delta)^2$ goes to infinity as $|\delta| \to \infty$, completing the proof. ■

Even if we proved that a solution exists, we cannot provide an explicit solution, therefore we rely on numerical solutions. However for the limiting case of firms being perfectly informed we are able to provide the exact solution. This is important since it highlights what is the main friction in setting monetary policy.

**Proposition 7** *If $P_\phi \to \infty$, then optimal monetary policy replicates the perfect information benchmark, that is $\delta^* = -1/\kappa$.*

**Proof.** When $P_\phi \to \infty$, firms only use their private signal, and therefore $\lambda_s \to 1$. Therefore the fixed point for $\alpha$ can be solved analytically and it implies $\alpha \to \frac{1-\omega\beta}{\omega}$. Also $\tilde{\rho} \to \frac{1-\omega\beta}{\omega}\kappa\delta\frac{P_\mu}{P_\mu+P_\theta}$ and $\varphi \to 0$. The minimization problem therefore becomes:

$$\min_{\delta}\left\{\left(\frac{1-\omega\beta}{\omega}+\frac{1-\omega\beta}{\omega}\kappa\delta\frac{P_\mu}{P_\theta+P_\mu}\right)^2\frac{1}{P_\theta}+\left(\frac{1-\omega\beta}{\omega}\kappa\delta\frac{P_\mu}{P_\theta+P_\mu}\right)^2\left(\frac{1}{P_\mu}+\frac{1}{P_\varepsilon\delta^2}\left(\frac{P_\theta+P_\mu}{P_\mu}\right)^2\right)\right\}$$

By taking out $\delta^2$ in the second term it is easy to show that the objective function is strictly convex in $\delta$ (it is a quadratic function), it therefore has a unique solution defined by the first order condition. Taking the derivative with respect to $\delta$ and simplifying we can show that it implies $\kappa\delta + 1 = 0$, giving the statement. ■

When agents are perfectly informed, monetary policy looses its informational value and retains only its allocational effect. Therefore the optimal choice of $\delta$ only implies offsetting the aggregate cost push shock, as in the perfect information benchmark economy. Note that this is so independently of the information precision of the central bank, $P_\mu$. The proposition highlights that the key informational friction for monetary policy is the imperfect information of the firms: when this are perfectly informed, the central bank should act as if it knew the shock. The implication is that it can be more important for a central bank to ascertain the amount of information among agents in the economy than understanding economic conditions itself.

## 6.1 Numerical Solution

We are interested in discovering some properties of the solution as function of structural information parameters (precisions of signals).[8] The rest of the parameters are set using standard in the literature. In particular a value of $\beta = 0.99$ is consistent with quarterly periods given an annual real interest rate of 3%. From the micro-foundations of the Calvo pricing model, $\kappa$ is given by the sum of the inverse elasticity of substitution of consumption

[8]We cannot prove that the objective is strictly convex in $\delta$ for a generic parameters' space, however in all our simulation loss function is always strictly convex in $\delta$, implying that the problem has a unique solution.

and of the inverse elasticity of substitution of labor. We consider a value of 1 for both these parameters so that $\kappa = 2$. $\omega$ is set to 0.5 implying that 50% of firms change their price every quarter, consistent with frequency of price adjustment in United States (Bils and Klenow 2004). Parameters values are summarized in Table (1). Before turning to the

| Parameter | Value |
|---|---|
| $\beta$ | 0.99 |
| $\omega$ | 0.5 |
| $\kappa$ | 2 |

Table 1: Non-information parameters value

actual result of the simulations, it is useful to state here what is the main finding and an intuition for it.

**Result 1 (Imperfect Information implies policy cautiousness)** *For any finite non zero values of precision of information parameters, the optimal monetary policy parameter $\delta^*$ is negative and $\delta^* > -1/\kappa$ ($\delta^*$ is lower in absolute value than optimal reaction with perfect information).*

The result of the numerical solution show that policy responses is more cautious than the perfect information benchmark: the optimal $\delta$ is lower in absolute value. The intuition of why it is optimal to have a more cautious policy is the following. As in all New Keynesian models, firms pricing decisions feature strategic complementarity since they depend on the aggregate price which is function of every other firm pricing decision. With strategic complementarities a public signal acts as a coordinating device on firms decisions, therefore firms rely heavily on it. In this model the information coming from the monetary policy rule is the public signal on which firms coordinate. Is the use of public information inefficient from the point of view of the central bank? The main objective of the Central Bank is inflation volatility stabilization, which ultimately depends on volatility of aggregate shocks and noise variables in the economy. When information in monetary policy is used by the firms, noise in central bank information increases volatility of inflation. On the other hand, private information of the firms does not have an effect on inflation volatility, since it is muted in the aggregate. Therefore the central bank has an incentive to decrease the informative role of monetary policy in order to dampen its effect on volatility of inflation and to foster less costly (always in terms of volatility of inflation) use of private sources of information. The way to decrease the informative role of monetary policy is to act more cautiously, effectively decreasing the precision of the information firms extract from the policy rule.

Figure 1 plots the optimal value of $\delta$ against $P_\varphi$, the precision of private information, while keeping all other parameters of precisions equal to 1. The simulations imply the following result:

**Result 2 (Optimal monetary policy as precision of private information changes)** *The optimal monetary policy is more cautious the less precise is private information of the firms.*

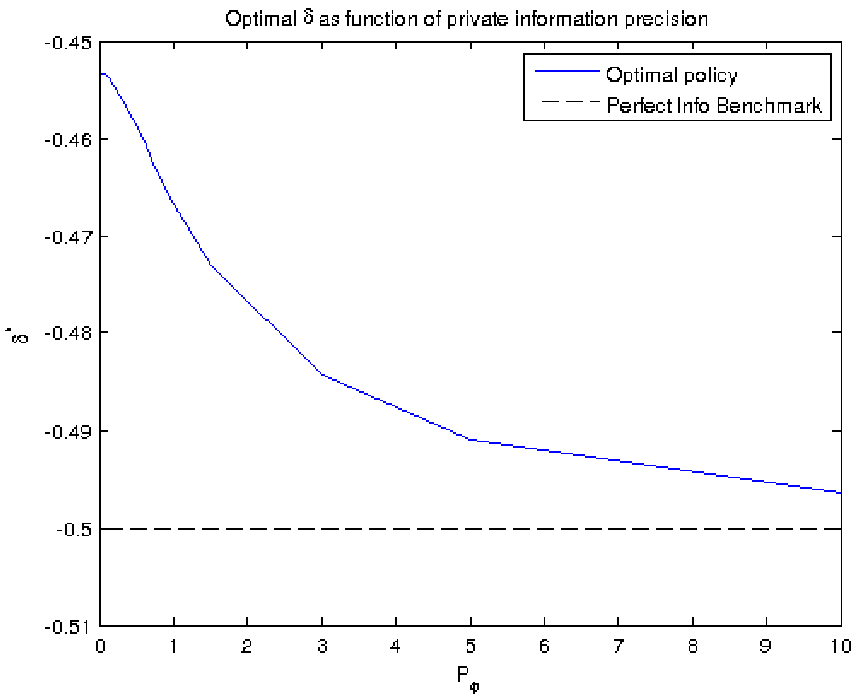


Figure 1: Optimal monetary policy as function of private information precision. All other information precision parameters set equal to 1.

The informational effect translates into inflation volatility through separate channels, and in order to better clarify the result it is useful to decompose equilibrium volatility of inflation (equation 44)) into three different components:

- $(\alpha + \tilde{\rho} + \varphi)^2 \frac{1}{P_\theta}$ represents the volatility generated by the cost push shock;
- $\tilde{\rho}^2 \frac{1}{P_x}$ represents the volatility generated by CB imperfect information and monetary policy shock;
- $\frac{\varphi^2}{(1-\omega)^2 \alpha^2 P_\eta}$ represents the volatility generated by the noisy price information.

Note in particular that, as we argued before, imperfect information of the central bank translates into a higher volatility of inflation. This is so since uncertainty in the central bank signal is transformed in a volatile output gap, increasing volatility of inflation.

In our simulations the important margins for determining the optimal $\delta$ are given however only by the first two components. These two margins are opposite in sign, but they push the optimal $\delta$ in the same directions. Lowering (in absolute value) $\delta$ is optimal either because in this way the central bank is making firms effectively use information, and therefore it lowers the contribution of the volatility of the cost-push shock on inflation. Or because it decreases the effect of distortions of its own imperfect information in the economy. Which of the two effects dominates depends overall precisions of information parameters' value.

With a very high value of private precision the certainty equivalence results of proposition (7) kicks in: optimal monetary policy is as close as possible to the perfect information benchmark. As private precision decreases, a lower (in absolute value) $\delta$ is optimal since, even though it increases the contribution of the cost push shock volatility, it decreases the effect of monetary policy distortions. However, as precision decreases even further, the reason for a decreasing (in absolute value) $\delta$ is reversed. Now by lowering $\delta$ the central bank fosters a more effective use of information decreasing the contribution of the cost-push shock volatility to aggregate inflation, while increasing the contribution of the noisy monetary policy.

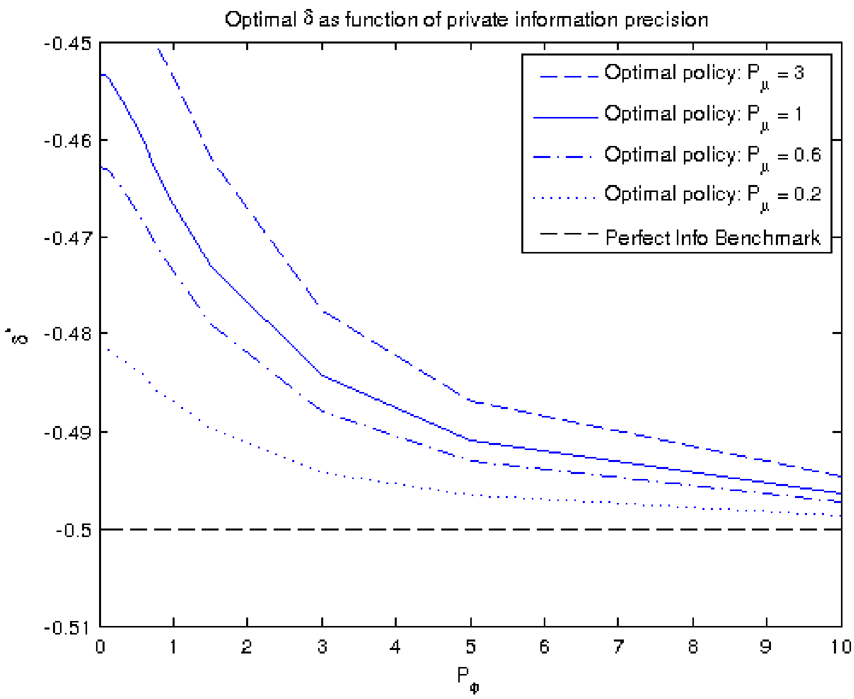


Figure 2: Optimal monetary policy as function of private information precision with varying central bank information precision.All other information precision parameters set equal to 1.

Figure 2 shows how optimal policy changes with respect to changes in the precision of central bank's information, which implies the following result:

**Result 3 (Optimal monetary policy as precision of central bank information changes)** *The higher the precision of the central bank information, the more cautious is the optimal* *monetary policy.*

When precision of central bank's information is very low, the informational effect of monetary policy is muted, therefore the best policy action is to follow the perfect information benchmark. When the precision of Central Bank's information increases, it increases the informational effect of monetary policy. With a more cautious monetary policy, the central bank is able to decrease firms' reliance on the information provided. As in the previous discussion, this can be optimal for two reasons: either because it decreases the pass-through of its imperfect information on inflation volatility, or because it foster more effective learning by the firms and so decreases the contribution of the cost-push shock volatility to inflation volatility.

The interesting result is that the more precise is the information available by the central bank information, the more cautious it has to be in reacting to the shock. This result can seem counter-intuitive, since more information of the central bank should imply a better knowledge of the economy and therefore a reaction closer to the perfect information benchmark. This line of reasoning is however not optimal in the present analysis since it does not consider how monetary policy is interpreted by firms, that is its informational effect. The more precise central bank's information, the more it is used by firms in their decision, reducing learning from other sources and increasing the effect of central bank's information on inflation volatility. Discarding considerations on how information in monetary policy affects firms' beliefs and decisions can therefore lead to higher volatility of inflation.

Figure 3 shows instead how the optimal policy changes with respect to changes in the precision of the price signal, determining the following result:

**Result 4 (Optimal monetary policy as precision of price signal changes)** *The less precise is the information transmitted by the price, the more cautious is optimal monetary policy.*

When the aggregate price communicates large amount of information, there is a decreased informational role of monetary policy, implying a reaction close to the perfect information benchmark. Note however that the amount of information transmitted by prices depends on the precision of private information ($P_\phi$), since the price aggregates the dispersed information in the economy. Even when the precision of the price signal is very high, if private information precision is low the price signal is not very informative, and therefore the informative effect of monetary policy is strong again. This implies the very steep change in optimal monetary policy we see in figure 3 when $P_\eta = 1000$ and private precision is low.

When the price signal is not very informative (or if in the limit there is no price signal at all) firms, missing a source of information, will heavily rely on the information provided

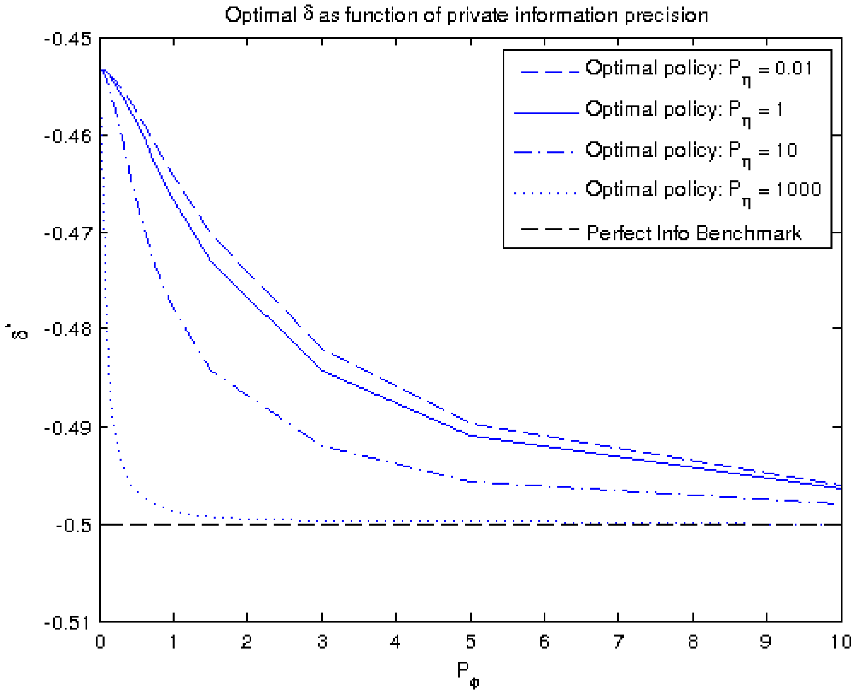


Figure 3: Optimal monetary policy as function of private information precision with varying price signal precision.All other information precision parameters set equal to 1.

by the monetary policy, that is there will be a very high informational effect. In order to minimize variance of inflation it is optimal for the central bank to undertake a more cautious policy reaction.

As we argued, the price signal can be interpreted as early information releases about inflation. This analysis highlights that monetary policy cannot leave out of consideration the level of information in the economy, whatever its sources might be, since the overall level of information is important in determining beliefs and actions of the firms, and consequently the best policy action.

# 7 Conclusion

This work introduces imperfect information in a simplified new-keynesian model and derives an additional role for monetary policy with respect to perfect information models, that of indirectly communicating to the agents the information available at the central bank. In this situation optimal monetary policy needs to take into account not only how it will directly influence the equilibrium inflation (the allocational role of monetary policy) but also how it will indirectly influence agents' beliefs and their pricing decision (the informational role of monetary policy). The main result is that imperfect information delivers policy cautiousness with respect to the perfect information benchmark economy:

monetary policy reacts less to the shock in order to not distort too much equilibrium volatility of inflation. On a general level, given the strategic complementarity of actions of the firms in the model, firms will internalize the public signal given by the monetary policy more than it is efficient for inflation volatility: by reducing the reaction to the shock the central bank is implicitly reducing the informative role of monetary policy. Specifically, noisy information in central bank information can influence both variability of equilibrium inflation and learning of the agents through other sources. We show that the two channels can have opposite effects on volatility of inflation depending on parameters values, however when discording roles, the marginal benefit given by decreasing the policy reaction to the shock is always greater than its marginal cost.

The implication of this work is that not only expectations about future shocks are important for monetary policy, but also the amount of knowledge about contemporaneous unobserved shocks. By influencing the beliefs of the agents in the economy monetary policy alters their learning process and can crowd out alternative source of information, that is the signaling role of prices. Monetary policy decisions should take into account the potential problem of focusing agents' decision only on the information provided by the central bank with the risk of inefficient outcomes for the economy. In this respect estimating the amount of knowledge of decision makers in the economy through surveys together with understanding how monetary policy will influence their beliefs is paramount for the work of a central banker in order to asses how the informational role and the allocational role of monetary policy will shape the outcomes of the economy.

The economic literature has only recently restarted to analyze the effect of imperfect information on monetary policy. This paper constitutes a contribution towards this goal, though many questions are still open. There are two lines of research stemming from this work. The first involves further exploring the signaling role of prices. In our model prices constitute a signal for the firms but not for the central bank. Allowing the possibility for the central bank to also observe prices generates a negative feedback effect for central bank learning: the higher the informational role of monetary policy, the less prices will communicate new information to the central bank, the more imperfect is its control of the economy. Optimal monetary policy in this aspect has to trade-off the need to provide information to the agents with the need for the central bank to efficiently learn. The second line of research takes into account the possibility for the central bank to directly reveal its information without communicating it indirectly through its policy, a transparency choice. As discussed in the related literature section, there is now a growing number of papers that explores this subject. However none, to the best of our knowledge, takes into account how transparency should balance the informational trade-off between public communication and learning from prices. Both these extensions will be explored in further research.

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